MECHANICS
FOR
JUNIOR
STUDENTS
BY

MECHANICS

FOR JUNIOR STUDENTS;

INCLUDING HYDROSTATICS AND PNEUMATICS.

BY

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PREFACE.

THIS work, originally adapted to the Elementary Examination of the Science and Art Department, has been greatly enlarged and in part rewritten, and it is hoped will prove deserving of a continuance of the favour with which it has hitherto been received. It is now suitable as a general text-book of Elementary Mechanics, and embraces the course for the Science and Art Department, for the Junior Examinations of the Universities, and for various Civi Service Examinations, as well as for the Examination of Teachers and Pupils of the Elementary Schools of Great Britain and Ireland.

The mathematical knowledge assumed on the part of the student extends only to the elements of geometry and algebra, and a few propositions of plane trigonometry. The Exercises have been selected chiefly from the Examination Papers of the Science and Art Department, and the Universities of London and Dublin.

The Sixth Edition has been carefully revised and enlarged, The Author begs to express his obligations to the Rev. J. Hamlyn Hill, M.A., Manchester, for much valuable assistance, by way of criticisms, suggestions, and additions.

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MECHANICS.

CHAPTER I.

MATTER, MOTION, AND FORCE.

1. **Matter.**—By the word matter we denote all things with whose properties we become acquainted by means of our senses. What matter is in itself we know not: we only know its effects, as revealed by our senses of sight, hearing, smell, touch, and taste.

2. A **Material Particle** is the smallest conceivable portion of matter, so small that its magnitude and form may be neglected, and its position alone considered.

3. A **Body** is a portion of matter limited in every direction, and having, therefore, a certain definite size and figure: it may be regarded as consisting of a collection of particles. A body is said to be *rigid* when all its particles are held together in an invariable position with respect to each other; *non-rigid* when this is not so. No body is perfectly rigid; but the particles of many bodies are so strongly united that we may treat the bodies as rigid.

4. **Motion** is change of place; its opposite is *rest.* A particle is at rest when it retains its position in space; in motion when it occupies different positions at different times. No body known to us is absolutely at rest. The earth, the sun, and all the heavenly bodies are in a state of perpetual motion; and bodies on the earth partake in all its motions. But if a body retains invariably the same position with reference to surrounding objects, it is at rest relatively to them. Thus, a ball lying on the floor of a moving railway carriage is at rest with relation to the other objects in the carriage, though all are really in motion with reference to objects on the railway. Such rest and motion may be distinguished as *relative.*

5. **Force** is any cause which produces or tends to produce a change in the state of rest or motion of a body. Thus a force may impart motion to a body originally at rest; it may

cause a body to move faster or slower in any given direction; or it may produce a change in the direction of motion.

6. All the forces with which we are acquainted arise from the action of one portion of matter upon another, in one of three ways:—

(1) By *pushing*, the bodies being in actual contact; in which case the force is called a *pressure* or *thrust*.

(2) By *pulling*, by means of some material connexion between them; in which case the force is called a *tension*.

(3) By some invisible influence, acting even when the bodies are at a distance from each other, and called an *attraction* or a *repulsion*, according as it pulls or pushes.

We have an example of pressure when we push a body with a stick; of tension, when we pull by means of a cord or of a rigid body; of attraction, when a needle is presented to the pole of a magnet; of repulsion, when the like poles of two magnets are brought near and drive each other away.

7. If only one force acts on a particle at rest, it must produce motion; if two or more forces act on a particle, they may or may not produce motion, each force in the latter case counteracting the effect of the remaining force or forces. When several forces act in this way on a particle or body without disturbing its condition, whether of rest or motion, they are said to be in *equilibrium*, or to *balance*.

8. **Equal Forces.**—Two forces which, when applied to a particle in opposite directions along the same line, balance each other, or produce equilibrium, are called *equal forces*. Thus a body suspended by a cord is kept at rest by two forces: its own weight, acting downwards, and the resistance exerted by the cord, acting upwards; and these two forces are equal. Hence—

(1) *Two forces which are equal and opposite are in equilibrium; and* (2) *Two forces which are in equilibrium are equal and opposite.*

9. If when two forces are applied to a particle in opposite directions along the same straight line, one overcomes the other and produces motion, the forces are *unequal*, and the former is said to be greater than the latter. If, again, two equal forces be applied to a particle along the same line and in the same direction, the result is a force double of either of the original forces; if three, treble; and so on.

10. *Any number of forces in equilibrium may be added to or taken from a system which is also in equilibrium, without disturbing the equilibrium of the system*; for, being in equilibrium, the forces so added or removed do not produce or tend to produce motion; hence their addition or removal can have no effect on the equilibrium of the system.

11. A force which acts only for an instant, the smallest conceivable period of time, so as merely to set a body in motion, is called an *impulsive* or *instantaneous force*; one which continues to act after setting the body in motion, is a *continuous force*; if it always produces the same effect in each unit of time, it is *constant*; if not, *variable*.

12. **Different States of Matter.**—Matter appears in three different states, according to the mode of aggregation of the particles. **A Solid** is a body whose particles retain the same relative positions with regard to one another when acted on by ordinary forces, being held together by mutual attraction or *cohesion*; ice, stone, wood, are examples. **A Liquid** consists of particles which have little or no cohesion, and can move easily among themselves, such as water. *Viscous* bodies, such as jelly and treacle, are intermediate between solids and liquids, and partake of the properties of both. An **Aëriform body** consists of particles which not only have no cohesion, but mutually repel each other. Hence, such bodies, when not acted on by any external force, tend to expand indefinitely. The smallest quantity of one of them will, under such circumstances, fill any vessel, however large, and exert a certain pressure against the sides.

13. Those aëriform bodies which retain that state at ordinary temperatures are called **gases**, such as the Oxygen and Nitrogen which compose the atmosphere; those which require high temperatures to retain them in the aëriform state are called **vapours**, as steam. As we shall not be much concerned with vapours here, we shall generally use the term *gas* instead of aëriform body, as more convenient.

Liquids and gases are included under the common name of **fluids**.

14. The same substance may exist in all three states at different temperatures and pressures. Thus the solid, *ice*, when heated becomes a liquid, *water*, and water still further heated, is changed into an aëriform body, *steam*. Probably every substance might be made to assume all three states.

Subjected to great cold and high pressure, all gases may be changed into the liquid or solid state, and all liquids into the solid state; and, on the other hand, every solid body could probably be reduced to the liquid state, and every liquid to the aëriform state, by the application of intense heat. Solids, liquids, and gases are therefore not different *kinds* of matter, but only different *states* in which matter appears.

15. **Mechanics** is that branch of science which treats of the laws of rest and motion of solid bodies, as produced by force. It is divided into

(1) *Statics*, which treats of forces in equilibrium, or balanced forces.

(2) *Kinematics*, which treats of motion, without reference to the bodies moving, or the forces causing motion.

(3) *Kinetics* or *Dynamics*, which treats of unbalanced forces, or forces producing motion.

Hydromechanics investigates in like manner the laws which govern the rest and motion of liquids and is divided into—

(1) *Hydrostatics*, treating of liquids at rest.

(2) *Hydrodynamics* (or *hydraulics*) treating of the motion of liquids.

Pneumatics is concerned with the laws of aëriform bodies. Hence Mechanical Science includes *Statics*, *Kinematics*, *Dynamics*, *Hydrostatics*, *Hydrodynamics*, and *Pneumatics*.

16. **Standards.**—In order to treat scientifically of forces and their effects, it is necessary to define the measures of time, space, and force to be employed.

Measure of Time.—Our measure of time is derived from the time of rotation of the earth on its axis before the sun. The average time of this rotation is called the *mean solar day*; it is divided into 24 hours, each hour into 60 minutes, and each minute into 60 seconds; and these divisions of time are measured by clocks and watches. The *unit of time* usually employed in works on Mechanics is the *second*, *i.e.*, the time occupied by one vibration of a pendulum, making 86,400 vibrations in the mean solar day.

The Unit of Length is the *Imperial Yard*; that is, the distance between two marks on the standard brass yard in custody of the Warden of the Standards, an officer of the

Board of Trade, when the temperature is 62° Fahrenheit, the barometer at 30 inches. *A Foot*, that is, one-third of a yard, is often taken as unit in Mechanics.

The Unit of Force is the *Pound Weight*, or *gravitation* unit, *i.e.*, the weight of the *Imperial Standard Pound*, also in the custody of the Warden of the Standards. This is the avoirdupois pound; it is divided into 16 ounces, and weighs *in vacuo* the same as 7,000 grains troy.

(Another mode of measuring a force—by the motion it produces—will be given further on.)

Accurate copies of the Imperial Yard and the Imperial Standard Pound are deposited in the Royal Mint, the Royal Society, the Greenwich Observatory, and the House of Parliament; and bronze copies are preserved in various other places; so that the standard measures can be replaced if lost, destroyed, or injured.

17. The following measures, which are frequently referred to, may be conveniently given here:—

At temp. 62° F., barometer 30 inches,

The Pendulum vibrating seconds at London is 39·1393 inches long.

A gallon of distilled water, containing 277·27 cubic inches, weighs 10lbs.

A pound of distilled water = 27·727 cubic inches.

A cubic foot of distilled water weighs 1000 oz. or $62\frac{1}{2}$lbs.

A cubic inch of distilled water weighs 252·456 grains. The more convenient number 252·5 will be used in the examples.

Circumference of circle $= 2\pi r$; Area $= \pi r^2$.

Surface of sphere $= 4\pi r^2$; Volume $= \frac{4}{3}\pi r^3$.

Where $\pi = 3{\cdot}1416$.

18. **Metric Standards.**—The French weights and measures are much more convenient than ours, being constructed on the decimal system. They are now usually employed in scientific treatises, and form the standards common to scientific men in all countries. They are founded on supposed natural measures; but their importance is quite independent of this derivation.

The unit of length is the **Metre**, which, when first adopted, was believed to be equal to the ten-millionth part of the meridian of Paris. The metre is subdivided into 10 decimetres, each decimetre into 10 centimetres, and each centimetre into 10 millimetres. Large measures are expressed by

multiples of the metre; thus 10 metres = 1 decametre; 100 metres = 1 hectometre; 1,000 metres = 1 kilometre. The centimetre and kilometre are the measures most commonly employed.

The **Unit of Surface** is the *square metre*, or **centiare.** 100 centiares = 1 are; 100 ares = 1 hectare.

The **Unit of Volume** is the cubic *decimetre*, or **litre.** The litre is subdivided into 10 decilitres, 100 centilitres, 1,000 millilitres; and the multiples of the litre are the decalitre, hectolitre, and kilolitre.

The **Unit of Weight** is the **gramme,** which is the weight at its maximum density (4° C.) of 1 *cubic centimetre* (called a millilitre) of distilled water. The gramme contains 10 decigrammes, 100 centigrammes, or 1,000 milligrammes; and its multiples are the decagramme, hectogramme, and kilogramme.

All these standards are constructed for the standard temperature 4° Centigrade, and standard barometric pressure of 760 millimetres.*

Platinum standards are carefully preserved in Paris for comparison.

19. *English and French Standards compared.*

0° C. = 32° F. 4° C. = 39·2° F.

760 millimetres = 29·9215 inches.

1 *Metre* = 39·3704 inches = 3·281 feet = 1·0936 yard.

1 *Yard* = ·9143 metre = 9·143 decimetres = 91·43 centimetres.

1 *Decimetre* = 3·937 inches, very nearly 4 inches.

1 *Centiare* (square metre) = 10·7641 sq. ft. or 1·196 sq. yds.

1 *Hectare* (100 ares) = 11960 sq. yds. = 2·471 (nearly 2½) acres.

1 *Square yard* = ·8365 centiare.

1 *Litre* = 1 cub. decimetre = 60·027 cub. in. (1¾ pints, very nearly)

1 *Kilolitre* (1,000 litres) = 216·4 gallons.

1 *Cubic foot* = 28·316 litres.

1 *Gallon* = 4·541 litres.

1 *Gramme* = 15·432 grains.

1 *Kilogramme* = 15,432 grains = 2·2 lbs. Avoirdupois.

1 lb. Avoirdupois = 453·6 grammes = ·4536 kilogramme.

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* The British Association Committee on Units recommend the adoption of the centimetre, gramme, and second as units for all scientific purposes. In this system, referred to as the C.G.S. system, the centimetre is taken as unit of length, the gramme as unit of mass, and $\frac{1}{g}$ gramme as unit of force, where $g = 981$ centimetres per second.

CHAPTER II.

PROPERTIES OF MATTER.

20. Certain properties are common to all substances, whether in a solid, liquid, or aëriform state: such as Impenetrability, Extension, Inertia: others belong to particular states, as Rigidity to solids; Fluidity to fluids.

21. **Extension.**—Every body occupies some space; the amount of this space is called its extension or *volume*; the boundaries constitute the *figure* of the body.

22. **Divisibility.**—It is now generally agreed that matter is not infinitely divisible; but that there is a limit beyond which it cannot be divided into smaller portions. The smallest or ultimate particles are called *atoms*: but the atoms are combined into collections of two or more, called *molecules*. These atoms and molecules are inconceivably minute—far beyond the reach of the most powerful microscope: nothing is known of their shape, but they are generally assumed to be spherical; and they are probably not in contact, but held at a distance from one another. They are subject to mutual attractions and repulsions, according to the state of the body. In solids they attract each other more or less strongly; in gases they repel; and in liquids the attractions and repulsions are equal, and balance each other.

23. The following examples will show how minute the molecules of bodies must be: A single grain of musk is said to have scented a room, with windows and doors constantly open, for ten years, yet at the end of that time was not perceptibly diminished in weight, though odorous particles or molecules must have been continually given off by it. The $\frac{1}{1000}$th part of a cubic inch of indigo will colour two gallons, or 555 cubic inches, of water. Dr. Wollaston obtained platinum threads $\frac{1}{3000000}$th of an inch in diameter. And the molecules of water are proved by chemists to be of such a size that 500 millions of them laid side by side in a straight line would not extend more than an inch.

24. **Porosity.**—On the molecular theory of matter, spaces must exist among the molecules of any body; these spaces

are called *pores*. They are, of course, invisible; but many bodies possess also sensible pores, such as some earthenwares; such bodies are called *porous*. On account of their porosity, all solids are permeable in a greater or less degree by liquids and gases.

25. **Impenetrability.**—Two portions of matter cannot occupy the same portion of space at the same time. When a nail is driven into wood, the particles of the wood separate, and the nail occupies the space between them. But, as we have just seen, no body quite fills up the whole space which it seems to occupy; and the molecules of one body may enter into the pores of another. Just as a vessel may appear filled with marbles, but may yet hold a quantity of shot poured in between the marbles, and afterwards a quantity of water, which will fill up the spaces between the solid bodies; so a cupful of tea may yet have a certain quantity of sugar added without overflowing; the particles of sugar separated by the solvent power of the liquid enter into the pores between the molecules of the latter. And a vessel apparently full of one gas may have a quantity of another gas passed into it without displacing the former. Hence impenetrability can only be considered as a property of the molecules of which bodies are composed.

26. **Compressibility** is the property of being reduced to smaller dimensions under increased pressure or diminished temperature, or both combined. Solids are usually only slightly compressible by the forces at our command—still such solids as caoutchouc are capable of great compression. Liquids also can be compressed to only a very slight extent; and hence they were formerly called incompressible fluids, under the belief that they did not admit of compression at all. Gases, on the contrary, admit of very great compression. Sir Isaac Newton believed it possible that the whole earth might be compressed into one cubic inch.

27. **Inertia** is the property in matter by which it resists any change of state. Thus a body cannot of itself change from rest to motion, or from motion to rest; some force is required to effect this change, that is, to overcome the *inertia* of the body. True, moving bodies often seem to stop of themselves. If a stone is thrown along the ground it soon comes to rest, but this is due to the friction against the ground. A stone thrown by the hand will shatter a pane of

glass, a bullet shot from a gun will cut away a circular portion. In the former case the impulse spreads itself over the whole pane, and every part yields more or less; in the latter there is not time to overcome the inertia of a larger part than that struck by the rapidly-moving body.

28. Attraction and Repulsion are properties universally inherent in matter. **Attraction** is the power by which every particle of matter draws or tends to draw towards itself every other particle. The particles of a solid, for example, are held together by the power of *cohesion;* the earth draws a stone towards it by the action of *gravitation;* a magnet attracts iron to itself by the power called *magnetism;* a glass rod rubbed with silk attracts a feather by the force of *electricity*. These are all different kinds of attraction. **Repulsion** is the opposite of Attraction. If water be sprinkled on a surface which has been rubbed with oil or grease the water is repelled, and does not moisten the surface. If the north pole of a magnet be held near the north pole of a magnetic needle it will repel the latter. Both attraction and repulsion increase with the nearness of the bodies. If two bodies attract each other with a certain force at any distance asunder, they will have *four times* as great attraction when brought to half the distance, *nine times* when brought to one-third the distance, and so on. This is usually expressed in the statement: *The attraction or repulsion between two bodies is inversely proportional to the square of the distance. Cohesion* is the mutual attraction between bodies of the same kind when their surfaces are brought in contact; this property is utilised in welding iron, preparing gutta-percha tubing, &c. *Adhesion* is the attraction between different kinds of bodies in contact, as solid and liquid.

29. **Hardness** is the resistance which a body offers to being scratched or torn by others, and depends on the mutual attraction of the molecules of the body. Diamond is the hardest body known; it scratches every other substance, but is not scratched by any. Wood is hard compared with clay, but soft compared with steel. Metals are often rendered harder by alloys of other metals: thus the silver used in coinage is hardened by being mixed with copper and the gold by being mixed with silver.

30. **Elasticity** is that property by which a body resists any change in its form or volume, and tends to recover these

when they have been forcibly altered. If a ball of indiarubber, ivory, or marble, be dropped on a smooth hard floor, it rebounds to a certain height. If it rises to the same height as that from which it fell its elasticity is said to be *perfect*, if to a less height, *imperfect*. If the ball be smeared with ink or paint it will leave on the floor a pretty large circle of the colouring matter, showing that when it struck the ground it became flattened, and then expanded so as to regain its shape. Part of the effect is due to the elasticity of the floor, which yielded to some extent at the time of the blow. If a ball of soft clay be dropped in the same way, it also will become flattened against the floor, but it will remain so, having scarcely any elasticity or tendency to regain its shape. The elasticity of the ball which rebounds is due to the repulsion developed among its particles by their being brought nearer than their natural distance. If this repulsion be greater than the cohesion by which they are held together the ball will be ruptured. The common idea, that the more easily anything stretches the more elastic it is, is exactly opposite to the mechanical idea of elasticity.

31. Most solids are very elastic for small changes of shape or volume, but if much distorted they acquire a set or permanent form, between the original and the distorted form. Liquids and gases resist change of volume only. Liquids, though usually called non-elastic fluids, are really very elastic; when compressed they regain their volume with great force; but only within so narrow limits that for all practical purposes they may be regarded as inelastic. Gases resist compression only; they expand spontaneously when pressure is removed. If perfect elasticity be denoted by 100, that of glass is 94; hard clay, 89; ivory, 81; marble, 79; hardened steel, 79; cast iron, 73; cork, 65; elmwood, 60; brass, 41; lead, 20; clay, 17.

32. **Resistance to Elongation.**—The elongation which a given force can produce in a bar or a beam depends on three things—the elasticity of the material of which the bar is composed, its section, and the stretching force itself. The material being the same, the greater the force the more will the bar be lengthened; the thicker the bar the less will be the elongation produced by any given force. In fact, *the elongation is directly proportional to the stretching force, and inversely to the section.*

33. If the length of a bar be L inches, and the area of its section A square inches, and if by the application of a strain of P lbs. its length be increased by l inches, it is found by experiment that $l : L :: \frac{P}{A} : E$, where E is a constant number, depending on the nature of the material, and is usually called the **Modulus of Elasticity.** Now, from this it appears that if $\frac{P}{A}$, that is, the strain per square inch, were equal to E, l would be equal to L—*i.e.*, if E lbs. were applied to each square inch of the section the length of the bar would be doubled. Hence

"*The Modulus of Elasticity is that strain per square inch of the section of a bar which would double its length if its elasticity remained perfect.*"

The elasticity in any case remains perfect within very narrow limits only. Beyond a certain limit the body regains its form less completely, and takes a set in its new form. This property is called the *ductility* or the *malleability* of the body, according as it is produced by traction or by hammering. It also ceases after a certain limit, depending on the tenacity of the body, and when this limit is reached rupture occurs. If the elasticity remained perfect under all circumstances, no change could be effected in the density or form of bodies.

34. The relations given in the last article may be more conveniently expressed in the form of an equation :—

$$PL = A\,l\,E.$$

Any four of these five quantities being given, the fifth may readily be found. In applying this equation, P must be expressed in lbs. when E is so expressed ; A in square inches, when E is expressed per square inch, L and l in the same units of length.

35. TABLE OF MODULI OF ELASTICITY.

Material.	*Modulus.*	*Material.*	*Modulus.*
	lbs..		lbs.
Wrought-iron bars.	29,000,000	Oak (English)	1,750,000
Cast Iron	17,000,000	Larch	1,360,000
,, Brass	9,170,000	Fir (Riga)	1,460,000
Steel (hard)	29,000,000	Elm	700,000
Copper Wire	17,000,000	Glass	8,000,000

The moduli given in this Table are average values derived from a number of experiments. Different specimens of any of the materials mentioned above may differ largely in structure, and hence give widely different results.

Example 1.—A bar of copper, 1 square inch in section and 20 feet long, is strained by a weight of 10 tons: how much is it lengthened ?

Here $P=22{,}400$lbs.
$L=240$ inches.
$A=1$ sq. inch.
$l=$required.
$E=17{,}000{,}000$lbs.

Therefore $22{,}400\times240=1\times l\times17{,}000{,}000$.

$$\therefore l=\frac{22{,}400\times240}{17{,}000{,}000}=\frac{5{,}376}{17{,}000}=\cdot3162 \text{ inch.} \quad \textit{Ans.}$$

Example 2.—By how much would a bar of wrought iron $\frac{1}{4}$ inch square and 100ft. long lengthen under a strain of two tons (neglecting the weight of the bar)?

$P=4{,}480$lbs.
$L=1{,}200$ inch.
$A=\frac{1}{16}$ sq. inch.
$l=$required.
$E=29{,}000{,}000$lbs.

$$\therefore 4{,}480\times1{,}200=l\times\tfrac{1}{16}\times29{,}000{,}000.$$

$$\therefore l=\frac{448\times12\times16}{29{,}000}=2\cdot966 \text{ inches.} \quad \textit{Ans.}$$

36 The modulus of elasticity of a liquid is the fraction which expresses the compression produced in a given bulk of the liquid by the application of a given force. At its temperature of maximum density (4° C., 39·2° F.) water undergoes a compression of ·00005 of its volume on the application of the pressure of one atmosphere, which may be called its modulus of elasticity.

37. **Tenacity** or **Resistance to Tearing.**—No solid substance can in reality bear a strain sufficient to double its length, *i.e.*, a strain represented by the modulus of elasticity. When the strain attains a certain magnitude the bar will break. The force required for each square inch of section to break the bar is called its tenacity. Thus for wrought iron the tenacity is 67,200lbs.; cast iron, 16,000lbs.; copper

wire, 60,000lbs.; iron wire ropes, 90,000lbs.; cast brass, 18,000lbs.

Here, calling the required force P, the tenacity T, we have

$$P = TA.$$

Example.—How great a strain will a cylindrical bar of wrought iron bear which is $\frac{1}{4}$ of an inch in diameter?

$$\text{Here, } P = 67{,}200\text{lbs.} \times A$$
$$A = \tfrac{1}{4}^2 \times {\cdot}7854 = {\cdot}04909.$$
$$\text{Hence } P = 67{,}200 \times {\cdot}04909 = 3{,}298{\cdot}68\text{lbs.} \quad \textit{Ans.}$$

38. **Endurance** is the resistance of a bar to a force which compresses it. When a bar is compressed in the direction of its length by a weight which is gradually increased, the bar gives way when the pressure reaches a certain amount. This crushing weight is regarded as a measure of the *endurance.* It is determined by experiment, and expressed in pounds per square inch. The endurance of cast iron is 112,000; wrought iron 40,000; cast brass 10,300; oak 10,000.

39. **Resistance to Bending** (flexual rigidity) depends, like the resistance to elongation, on the force applied, the section, and the modulus of elasticity. It is proportional to the breadth of the bar or beam, and to the *square* of the depth. Hence joists are laid, not flat, but on their side; their strength depends more on their depth than on their breadth.

40. **Resistance to Twisting** is measured by suspending a wire or rod of the material vertically, the upper end being rigidly fixed, and the lower end having a needle fixed at right angles to the rod. When the wire is twisted, the needle indicates on a graduated scale below the angle through which it has been turned; this is called the *angle of torsion*; and the force required to retain the needle in that position is the *force of torsion* for that particular angle. "Generally speaking, the resistance of a cylinder, solid or hollow, to wrenching, is double of its resistance to breaking across."—(Rankine, 355).

41. All circumstances which interfere with the molecular condition of bodies modify their elasticity. Sometimes the elasticity increases when the temperature increases, but very frequently it decreases. No constant laws have yet been determined.

42. **The Dynamometer** (*dynamis,* force; *metreo,* I measure, G.), an instrument used to measure force, depends on

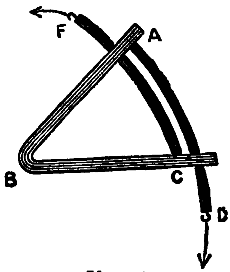

FIG. 1.

elasticity. It is of various forms, one of which is represented in Fig. 1. A B C is a strong steel spring. The circular arc A D is attached to the arm A B at A, and passes freely through a slit in B C. In the same way C F is fixed at C, and passes freely through B A. If we want to measure the force applied, say, to move a carriage, let the carriage be attached to D, and the horse to F. The force applied will be shown by the extent to which the arms C and A approach, as indicated on the graduated arc A D or C F.

43. The **Spring Balance**, a kind of dynamometer, consists of a spiral spring, which is elongated by the weight attached to it, and forms a convenient instrument for comparing the weight of the same body at different latitudes, and for showing the varying amount of gravity. Its efficiency depends on the torsional rigidity of the wire composing it. The ordinary balance is a measure of *mass*, because it always gives *in vacuo* the same indication with the same body; but the spring balance is really a measure of *force*; and hence the same body will affect it differently according as it is tried at the equator, at London, or at the pole; because the force of gravity varies at these places. The ordinary letter-weigher is a spring balance.

44. **Moduli on Metric System.**

	Kgs. per sq. mm.
Flint Glass..................	5851
Brass	10948
Steel	21793
Wrought Iron	19994
Cast Iron	13741
Copper	12558

EXERCISES.

1. The modulus of elasticity of a substance is 30,000,000: under what tension will a rod 30 feet long and 5 square inches in section be stretched $\frac{1}{4}$ of an inch?

$$PL = A\,l\,E.$$

$$\therefore P \times 360 = 5 \times \tfrac{1}{4} \times 30{,}000{,}000.$$

$$\therefore P = 150{,}000{,}000 \div 1440 = 104{,}166\tfrac{2}{3}\text{lbs.} \quad \textit{Ans.}$$

2. The modulus of elasticity of copper wire being 17,000,000, determine the elongation of a wire 12 feet long, whose section is $\frac{1}{100}$ square inch, under a pressure of 100lbs. *Answer*, $\frac{36}{425}$ inch.

3. If 12 iron wires, each $\frac{1}{10}$ inch in diameter, just sustain a weight of 3 tons, what is the *tenacity* of iron wire? *Answer*, 31·831 tons, or 71,301·24lbs.

4. What strain will be borne by a bar of cast iron whose section is a square $\frac{2}{3}$ of an inch in the side?

$$P = TA = 16{,}500 \times \tfrac{4}{9} = 7{,}333\tfrac{1}{3}\text{lbs.} \quad \textit{Ans.}$$

5. A cylindrical brass rod is one inch in diameter; what weight will it bear without being crushed?

$$P = AC = 3{\cdot}1416 \times \tfrac{1}{4} \times 10{,}300 = 8{,}089{\cdot}62\text{lbs.} \quad \textit{Ans.}$$

6. A rod 30 feet long, and 3 inches by 4 inches in section stretches $\frac{1}{10}$ inch under a force of 6 tons: what must be the length of a rod of the same material 2 inches square, which stretches $\frac{1}{3}$ inch under a force of 20 tons?

First Rod.	*Second Rod.*
$P = 13{,}440$lbs.	$P = 44{,}800$lbs.
$L = 360$in.	L required.
$A = 12$ sq. in.	$A = 4$ sq. in.
$l = \frac{1}{10}$in.	$l = \frac{1}{3}$in.

Hence, in first rod $PL = A\,l\,E$.

$$\therefore 13{,}440 \times 360 = 12 \times \tfrac{1}{10} \times E.$$

$$\therefore E = \frac{13{,}440 \times 360 \times 10}{12} = 4{,}032{,}000\text{lbs.}$$

Since the material is the same in both rods, E is constant. In the second rod, $PL = A\,l\,E$.

$$\therefore 44{,}800 \times L = 4 \times \tfrac{1}{3} \times 4{,}032{,}000.$$

$$\therefore L = \frac{4 \times 4{,}032{,}000}{3 \times 44{,}800} = 120 \text{ inches} = 10 \text{ feet.} \quad \textit{Ans.}$$

7. Find the elongation produced on a bar of copper 10 metres long and 1 square mm. in section by a weight of 500 kilogrammes.

$$PL = A\,l\,E \;\therefore\; 500 \times 10{,}000 \text{ mm.} = 1 \times l \times 12{,}558.$$

$$\therefore l = 5{,}000{,}000 \div 12{,}558 = 398{\cdot}9\text{mm.} \quad \textit{Ans.}$$

8. What weight will be required to stretch a steel wire 20 metres long, diameter 5mm., by 1 metre?

Here $PL = AlE \;\therefore\; P \times 20 = 5^2 \times {\cdot}7854 \times 1 \times 21{,}793.$

$$\therefore P = 25 \times {\cdot}7854 \times 21{,}793 \div 20.$$

$$= 21395{\cdot}27775 \text{ kg.} \quad \textit{Ans.}$$

CHAPTER III.

COMPOSITION OF FORCES.

45. The elements required for the specification of a force are:—

(1) *Its point of application, i.e.*, the position of the particle on which it acts; or the point of the body to which it is applied.

(2) *Its line of action, i.e.*, the straight line along which it tends to move the particle or body.

(3) *The direction* in which it acts along the line of action.

(4) *Its magnitude or intensity.* This is measured by the weight which it could support in opposition to gravity. A force which could just support a pound of matter is called a force of 1lb.; a force which could support 10lbs. is ten times as great, and is called a force of 10lbs. Any force is said to be known or given when these four elements are known.

46. Forces may be conveniently represented by lines. The point where a line representing a force begins denotes the point of application of the force; the direction of the line marks the direction in which the force acts; and the number of units of length in the line denotes the number of units of weight in the force. Thus, in Fig. 2, the lines A B, A C, represent forces of 3lbs. and 5lbs. respectively, acting at the point A in the directions from A to B, and from A to C. The direction is indicated by the order in which the letters are taken, or by a barb or arrow in the diagram.

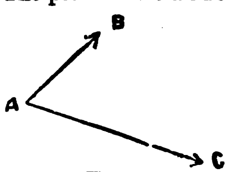

FIG. 2.

47. **Transference of Forces.**—A force acting at any point may be considered as applied in the same direction at any other point along its line of action, provided the new point be rigidly connected with the former. Thus, a push transmitted by means of a rigid rod will have the same effect whatever be the length of the rod; and a pull exerted through a string will be the same in effect at whatever point of the string it be

applied. This principle is constantly assumed throughout the whole science of statics, and the assumption is fully warranted by results. Conversely, if a force can be transferred from its point of application to any other point without altering its effect, that point is in the line of action of the force.

48. **Composition and Resolution of Forces.**—If two or more forces act at the same time on any body, their effect is the same as that of some single force, which might therefore replace them; such a force is called their *Resultant*, the original forces being the *Components*. Given the components, the process of finding the resultant is called *Composition of Forces;* the converse process of finding components equivalent to a given resultant is called *Resolution of Forces.*

SECTION I.

FORCES ACTING ON A PARTICLE.

Forces acting on a particle may have the same line of action, or their lines of action may intersect.

49. **Forces acting along the same line.**—If two equal forces act on a particle in opposite directions along the same line, the particle will not move—the forces will be in equilibrium, and the resultant will be zero. If two forces act in the same direction along the same line, the resultant will evidently be the sum of the forces in the same direction. If two unequal forces act in opposite directions, the resultant will be their difference, in the direction of the greater force. Hence, the resultant of two forces acting along the same line can never exceed their sum, or be less than their difference.

50. If several forces act in one direction, and several in the opposite along the same line, the resultant is equal to the algebraic sum of the components. It is convenient to consider the forces acting in some one direction as *positive*, and those acting in the opposite direction as *negative*. Thus, suppose forces of 24, 17, and 41 pounds act on a particle towards the north, and forces of 7, 20, 16, and 10 towards the south, the resultant will be the algebraic sum, viz.,

$24+17+41-7-20-16-10=29$lbs. towards the north.

In our diagrams, forces represented as acting towards the top or towards the right hand side of the page will be regarded

as positive, and characterised, when necessary, by the sign +. Those acting towards the bottom or towards the left hand side of the page as negative, with the sign −.

51. If a particle be kept at rest by several forces, it is plain that each force balances all the other forces, or their resultant. *Each force is therefore equal and opposite to the resultant of all the others.*

52. **Intersecting Forces.**—If two forces act along lines which meet at a point, their resultant lies between their directions and in their plane. Suppose a particle at A (Fig. 3), acted on at the same time by a force A B, pulling it in the direction from A to B, and a force A C pulling it in the direction of C, the particle will tend to move neither along A B nor A C, but along some intermediate line A D, whose direction will depend on the relative magnitudes of the two forces. If the forces be equal, this line will bisect the angle between A B and A C, for there is no reason why it should lie nearer to one than to the other. If the forces be unequal, this line will be nearer to the line of action of the greater force. The exact position of the line A D can be found by an easy geometric construction. Let A B, A C (Fig. 3) represent the forces; complete the parallelogram A B, C D, and then draw the diagonal A D; A D will be the resultant both in magnitude and direction. The proposition embodying this fact is the fundamental principle of statics, viz.—

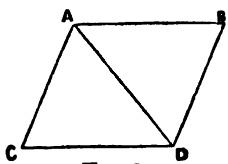

FIG. 3.

53. **Parallelogram of Forces.**—*If two forces acting on a particle be represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant shall be represented, in magnitude and direction, by the diagonal drawn through the point in which those sides meet.*

This proposition is true of all forces, whether producing motion or not. It can be proved mathematically, but the demonstration is too long and complicated to introduce here.

54. *Experimental Proof.*—Suspend three weights, any two of which are together greater than the third—say 9, 11, and 6lbs. respectively—over two smooth pegs or pulleys, as in Fig. 4, by three fine strings knotted together at A, and allow the

system to adjust itself in a position of equilibrium. In the position represented by the figure, the weight of 11lbs. is sustained by the joint action of the two, 9lbs. and 6lbs., or by their resultant. It is therefore equal and opposite to their resultant (Art. 51), and hence the resultant acts vertically upwards through A. Make A D=9 units of length, A B=6, and complete the parallelogram A B C D. Then A C, the diagonal, will be found to be vertical and 11 units in length. Therefore A C represents the resultant both in magnitude and direction. The same will hold good whatever be the weights we employ. The pegs or pulleys over which the cords pass are supposed to be perfectly smooth, and therefore do not affect the result. If the components are equal, the line A C will bisect the angle B A D; if they are unequal, the resultant will always be nearer the greater component, as in the diagram.

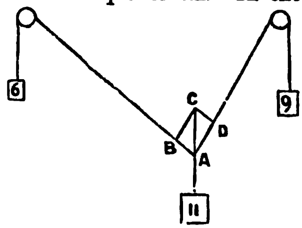

FIG. 4.

55. To determine the resultant of two intersecting forces represented by A B and A C (Fig. 5), the following is the simplest practical method:—Join B C, and bisect it in the point E; join A E, and produce it to D, making E D=A E. Then A D represents the resultant.

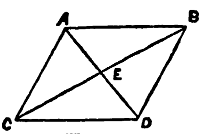

FIG. 5.

56. If three or more forces act on a point the resultant is found by a continuation of the same process. We find the resultant of two of them; then of this resultant and a third force; of this resultant and a fourth force, and so on.

57. Innumerable illustrations of the composition of forces might be given. When a horse draws a boat along a canal the horse pulls the boat in a slanting direction towards the bank along which he is walking, the helm tends to turn it towards the opposite shore; it obeys neither, but moves between in a diagonal direction. The flight of a bird is another example; the stroke of each wing urges the bird in a different direction, but it moves between those directions. The flying of a kite again depends on the combined effect of the cord, the wind, and the weight of the kite.

58. The following examples will illustrate the numerical calculation of the resultant of two forces acting at a point in a few simple cases. The components are represented by P and Q, the resultant by R.

The resultant of two forces acting at an angle is always less than their sum, and greater than their difference. For, in Fig. 7, A C+C D are >A D and C D=A B $\therefore$ P+Q>R. Again A D+D C are >A C $\therefore$ R+P>Q $\therefore$ R>Q−P.

Examples.

(1) Two equal forces of 10lbs. each act on a particle at right angles: find the resultant. Here the forces are represented by two sides of a square, and the resultant by the diagonal, $\therefore R^2=10^2+10^2=200$, and R=14·14lbs.

(2) Two forces of 30lbs. and 40lbs. act on a particle at right angles: find the resultant. The forces are represented by the sides A B, A C (Fig. 6), and the resultant by the diagonal A D. Now A D^2=A B^2+B $D^2=30^2+40^2=2500$. Therefore the resultant is a force of 50lbs. in the direction A D.

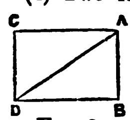

FIG. 6.

(3) Two equal forces of 50lbs. act on a particle at an angle of 120°: find the resultant. Here the forces are represented by two sides of a rhombus, and the resultant by the shorter diagonal. This diagonal is easily proved geometrically to be equal to a side, and therefore the resultant is of the same magnitude as each component, *i.e.*, 50lbs.

(4) Forces of 30lbs. and 40lbs. act on a point at an angle of 60°: find their resultant. The forces are represented by the sides A B, A C (Fig. 7), and the resultant by the diagonal A D. Produce A B to meet a perpendicular drawn to it from D, in F. Then since D B F is 60°, the triangle D B F is half of an equilateral triangle. Therefore B F=$\frac{1}{2}$B D=20. Then D F^2=B D^2−B $F^2=40^2-20^2=1200$. Now A D^2=D F^2+A F^2=1200+2500=3700 $\therefore$ A D=60·8. *Ans.*

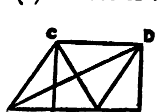

FIG. 7.

(5) Forces of 30lbs. and 40lbs. act at an angle of 120°. Here resultant=B C=$\sqrt{G\ C^2+G\ B^2}$ (Fig. 7). B G=A B−A G=10 $\therefore$ B C=$\sqrt{1200+100}=\sqrt{1300}$=36·05lbs. *Ans.*

(6) Let the forces be 30lbs. and 40lbs., acting at an angle of 30°; and let them be represented by A B, A C (Fig. 8); then A D will represent the resultant. Draw D F perpendicular to A B produced, to meet it in F. Then B D F is half of an equilateral triangle, and D F is $=\frac{1}{2}$ B D=20. $\text{B F}^2=\text{B D}^2-\text{D F}^2=40^2-20^2=1200$

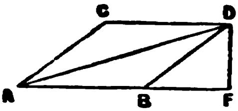

FIG. 8.

$\therefore \text{B F}=\sqrt{1200}=34{\cdot}641.$ $\text{A D}^2=\text{A F}^2+\text{D F}^2=400+4178{\cdot}4 \therefore$ A D=67·66. *Ans.*

(7) Let A B=30, A C=20, B A C=45° (Fig. 9). $\text{B F}=\text{D F}=\sqrt{\text{B D}^2\div 2}=14{\cdot}142 \therefore \text{A D}^2=\text{A F}^2+\text{D F}^2=(44{\cdot}142)^2+(14{\cdot}142)^2=1948{\cdot}516+200=2148{\cdot}516 \therefore$ A D=46·35lbs. *Ans.*

59. We shall now deduce the very important formula, which, with the aid of a simple table, enables us to calculate the resultant of any two forces acting at any given angle: In Fig. 9, the angle B A C being acute $\text{A D}^2=\text{A B}^2+\text{B D}^2+2\,\text{A B.B F}.$ (Euc. II., 12.)

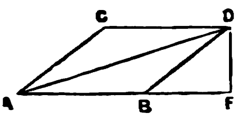

FIG. 9.

Now $\frac{\text{B F}}{\text{B D}}$ is the cosine of D B F or of B A C.

$\therefore$ B F=B D cos. B A C=A C cos. B A C.

Therefore, $\text{A D}^2 = \text{A B}^2 + \text{A C}^2 + 2\,\text{A B.A C} \cos. \text{B A C}.$

Again, in Fig. 9A, when the angle B A C is obtuse, we have (by Euc. II., 13) $\text{A D}^2=\text{A B}^2+\text{B D}^2-2\,\text{A B.B F}.$

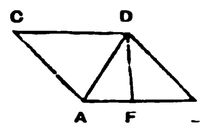

FIG. 9A.

Now $\frac{\text{B F}}{\text{B D}}=$cos. D B F=−cos. B A C.

$\therefore$ B F=−B D cos. B A C.

$$\therefore \text{A D}^2=\text{A B}^2+\text{B D}^2-2\,\text{A B}\times(-\text{B D} \cos. \text{B A C}).$$
$$=\text{A B}^2+\text{B D}^2+2\,\text{A B.B D} \cos. \text{B A C}.$$
$$=\text{A B}^2+\text{A C}^2+2\,\text{A B.A C} \cos. \text{B A C}, \text{ as before.}$$

Let A B be called P, A C, Q, and A D, R; and let B A C be called $\widehat{PQ}$. Substituting these values, we have

$$\mathbf{R^2 = P^2 + Q^2 + 2\,PQ \cos. \widehat{PQ}}.$$

60. *Table of cosines required in the Examples, &c.*

Cos. $30° = \frac{\sqrt{3}}{2} = {\cdot}866.$ Cos. $150° = -\cos. 30° = -{\cdot}866.$

$\text{Cos. } 45° = \frac{1}{\sqrt{2}} = \cdot 707107.$ $\text{Cos. } 135° = -\text{cos. } 45° = -\cdot 707107.$

$\text{Cos. } 60° = \frac{1}{2} = \cdot 5.$ $\text{Cos. } 120° = -\text{cos. } 60° = -\cdot 5.$

$\text{Cos. } 90° = 0.$

$\text{Cos. } 0° = 1.$ $\text{Cos. } 180° = -\text{cos. } 0° = -1.$

The student will note that the greater the angle the smaller the cosine; hence the resultant increases as the angle between the forces diminishes.

61. The examples of Art. 58 are here worked by the formula.

$$R^2 = P^2 + Q^2 + 2\,PQ \text{ cos. } \widehat{PQ}.$$

(1) $R^2 = 10^2 + 10^2 = 200 \therefore R = \sqrt{200} = 14 \cdot 14$lbs. *Ans.*

(2) $R^2 = 30^2 + 40^2 + 2400 \times 0 = 2500 \therefore R = 50$lbs. *Ans.*

(3) $R^2 = 50^2 + 50^2 + 2 \times 50 \times 50 \times -\cdot 5 = 5000 - 2500 = 2500.$

$\therefore R = \sqrt{2500} = 50$lbs. *Ans.*

(4) $R^2 = 900 + 1600 + 2400 \times \cdot 5 = 3700 \therefore R = \sqrt{3700} = 60 \cdot 8$lbs. *A.*

(5) $R^2 = 900 + 1600 + 2400 \times -\cdot 5 = 1300 \therefore R = 36 \cdot 05$lbs. *Ans.*

(6) $R^2 = 900 + 1600 + 2400 \times \cdot 866 \therefore R = \sqrt{4578 \cdot 4} = 67 \cdot 66$lbs. *Ans.*

(7) $R^2 = 900 + 400 + 1200 \times \cdot 7071 = 2148 \cdot 52 \therefore R = 46 \cdot 35$lbs. *Ans.*

62. **Triangle of Forces.**—*If three forces, acting on a particle, keep it at rest, they may be represented by the three sides taken in order of any triangle, whose sides are parallel to their several directions.* Thus if a particle B be kept at rest by the three forces B D, B C, B G, it is plain that if B G were removed the particle would be acted on by the resultant of B C, B D (Fig. 10). B G, therefore, must be equal and opposite to that resultant. Complete the parallelogram B C F D. B F is the resultant $\therefore$ B F = B G $\therefore$ F B = B G; and B C = D F. Therefore the three given forces may be represented by the sides of the triangle B D F, viz., B D, D F, F B. This proposition supplies an easy method of finding the resultant of two intersecting forces. Thus if B C and B D represent the forces, we have only to draw B D in the direction of its force, then from D draw D F in the proper direction and equal to B C, and join F B. F B denotes the force which would balance B D, B C; therefore B F denotes their resultant. The converse of this—*If three forces, acting on a particle, be represented in magnitude and direction by the sides of a triangle,*

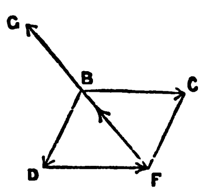

Fig. 10.

taken in order (i.e., in a direction going continuously round the triangle) they are in equilibrium—is also true, and may be easily proved from the same figure.*

63. **Polygon of Forces.**—In a similar way it can be shown that *if any number of forces, in the same plane, acting at a point, are in equilibrium, they may be represented, in magnitude and direction, by the sides of a polygon taken in order.* For, combining any two of the forces by the triangle of forces, say A B, B C (Fig. 11), we see that the resultant is A C. The resultant of A C and C D is A D, and that of A D and D E is A E. The resultant of all the forces A B, B C, C D, D E, therefore, is A E. This will be balanced by the force represented by E A, the remaining side of the polygon; so that if the forces be in equilibrium they may be represented by all the sides of the polygon taken in order. Hence, if all the sides of a polygon except one, taken in the same order round the figure, represent any number of forces acting at a point, the remaining side, taken in the opposite direction, will represent the resultant. Also, conversely, *If all the sides of a polygon, taken in the same order round the figure, represent, in magnitude and direction, a number of forces acting at a point, those forces are in equilibrium.*

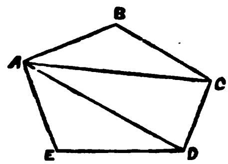

FIG. 11.

64. **Resolution of Forces.**—A line may be made the diagonal of an infinite number of parallelograms. Hence, since any line may represent a force, any force may be the resultant of an infinite number of pairs of components. Thus in Fig. 12 A D may be the resultant, either of A B, A C, or of A F, A G, or of some other pair of forces. Hence we see that though any two forces given, in magnitude and direction, can have only one resultant, yet any given resultant can have many sets of components.

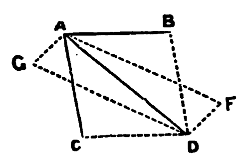

FIG. 12.

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* Since the sides of a triangle are proportional to the sines of the opposite angles, if three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two, *i.e.*,

$$P : Q : R :: \sin.\ \widehat{QR} : \sin.\ \widehat{PR} : \sin.\ \widehat{PQ}.$$

65. To resolve a force into two separate forces we have only to draw from one of the extremities of the line representing it two straight lines in any directions whatever making an angle with each other, one being on each side of the given line, and making with it angles which are together less than two right angles; then through the other extremity draw two lines parallel to the former two. The adjacent sides of the parallelogram so formed will represent, in magnitude and direction, the required components.

66. The commonest case of resolution is that in which it is to be ascertained what portion of a force is effective in a direction different from its own. A force has no effect whatever in a direction at right angles to its own. Suppose we now wish to resolve the force represented by A B (Fig. 13), into the parts effective and not effective in a horizontal direction. From A draw A C horizontal and A D vertical, and complete the parallelogram A C B D. Then A C represents the portion really useful in the horizontal direction. A D is effective only in a vertical direction, *i.e.*, it is expended in lifting the body it is applied to. Suppose A B to represent a road, up which a horse exerts a pull of 1 ton. If B C be 1 foot, A B 6, one-sixth of the pull is expended in lifting the load.

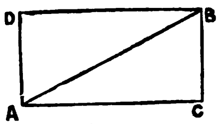


FIG. 13.

67. When a force is *resolved in a given direction*, it is resolved into two components, one in the given direction, the other at right angles to it. *The resolved part of the resultant of two forces in any direction is equal to the sum of the resolved parts of the components in the same direction.* Let the forces A B and A C (Fig. 14), and their resultant A D be resolved along A E, and at right angles to it. The vertical component of A D is D E; of C A, or which is the same, of B D, the vertical component is D F, and of A B, F E. Now D E is the sum of D F and F E. Again, the horizontal component of A D is A E, and those of A B and A C or B D, are A G and G E, which together make up A E. Therefore the proposition is established.

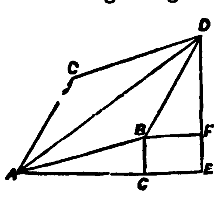


FIG. 14.

*68. **Method of Co-ordinates.**—The method of Art. 67 may be applied to find the resultant of any number of forces in one plane meeting at a point, by taking any line through the point, resolving each force along this line and at right angles to it, taking the sum of the components in each direction as the component of the resultant in that direction, and compounding the components so found.

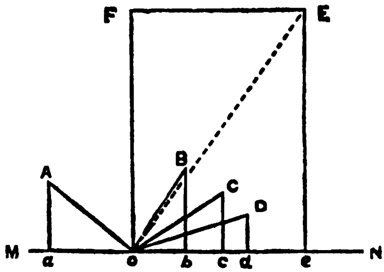


FIG. 15.

For example, suppose the particle O to be acted on by the forces O A, O B, O C, O D, as represented in Fig. 15. Through O draw any straight line M O N; from A, B, C, D draw perpendiculars to M O N. Then the components of O A are a A and $-$ O a; of O B, b B and O b; of O C, c C and O c; of O D, d D and O d. Hence, adding, we have along the line O N a force$=$O $b+$O $c+$O $d-$O a, and at right angles to it a force $=a$ A$+b$ B$+c$ C$+d$ D. Along O N mark off O $e=$O $b+$O c $+$O $d-$O a; from e draw e E at right angles to O e, and$=$ a A$+b$ B$+c$ C$+d$ D; join O E; then, by the triangle of forces, O E represents the resultant of O A, O B, O C, O D. O a and a A are called, in geometry, the *co-ordinates* of O A; O b and b B the co-ordinates of O B, &c.

*69. **Trigonometrically.**—Let P, Q, R . . . denote the forces . . . let α β γ . . . denote the angles which their directions make with a fixed straight line, A x, drawn through the particle A on which the forces act; and let the line A y be drawn through A at right angles to A x.

P may be resolved into		P cos. α along	A x, and	P sin. α along	A y.
Q	„	„	Q cos. β „	„	Q sin. β	„
R	„	„	R cos. γ „	„	R sin. γ	„

and so on for all the forces. Adding these components we have

$$P \cos. \alpha + Q \cos. \beta + R \cos. \gamma + \ . \ . \ . \ \text{along A } x.$$
$$P \sin. \alpha + Q \sin. \beta + R \sin. \gamma + \ . \ . \ . \ \text{along A } \gamma.$$

Call the former sum X, the latter Y. Hence the given forces

are equivalent to the forces X and Y in directions which are at right angles to each other. Let K be their resultant, and θ the angle which the direction of K makes with A x. Then $K^2=X^2+Y^2$ . . . K cos. $\theta=X$ . . K sin. $\theta=Y$. Thus the resultant is determined in magnitude and direction.

70. **Conditions of Equilibrium**:—

(1) *Of two forces acting on a particle.* The forces must be equal in magnitude, and must act along the same line, in opposite directions.

(2) *Of three forces acting on a particle.* *a.* Each force must be equal and opposite to the resultant of the other two. *b.* The forces may be represented by two adjacent sides of a parallelogram and the diagonal taken in the reverse direction. *c.* They may be represented by the sides of a triangle taken in order.

(3) *Of any number of forces acting in one plane on a particle* The forces will be in equilibrium if their resultant vanishes, *i.e.*, if K=O; and this will be the case if X=O, and Y=O. Hence the condition of equilibrium is, that the sum of the resolved parts of the forces along each of two straight lines at right angles to each other vanishes.

71. Forces which act at various points in one plane, and which are not parallel, can be compounded by a repetition of the processes of composition and transference. Thus, if P_1, P_2, P_3 be three forces acting at different points in the same plane: Transfer P_1 and P_2 to the point at which their lines of action meet, and find their resultant; transfer this resultant and P_3 to the point at which their lines of action meet, and find their resultant; this will evidently be the resultant of all the three forces; and so with four or more forces.

72. The student should accustom himself to work out problems by means of diagrams drawn to scale. For this purpose he will require parallel rulers, a graduated scale, a pair of compasses, and a protractor for marking off angles. The larger the scale on which a diagram is drawn, there is the less chance of error in the results obtained from it.

In working a problem by means of a diagram, the following points should be attended to:—

(1) The scale should be shown, with the statement, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{m}$, &c., inch to the pound, as the case may be.

(2) The geometrical construction should be written out as in a proposition in Euclid.

(3) The mechanical statement should be given, viz., that by the parallelogram, or triangle, or polygon of forces, certain results follow.

(4) A statement of the measurements obtained.

EXERCISES.

1. The greatest and least resultants of two forces acting on a particle are 100 and 30: find them. *Answer.* 65 and 35.

2. Two forces of 24lbs. and 36lbs. act on a particle at right angles: find their resultant. *Answer.* 43·26lbs.

3. The resultant of two forces acting at right angles is 100lbs., and one force is 60lbs.: find the other. *Answer.* 80lbs.

4. The resultant of two equal forces acting at a point in directions at right angles is 50lbs.: find them. *Answer.* 35·35lbs. each, making angles of 45° with the resultant.

5. If two forces of 5lbs. and 7lbs. act at a point in directions at right angles, show by a construction how they act.

6. Forces represented by 1, 2, 3, 4lbs., taken in order, act at a point in directions at right angles to each other: find the magnitude and direction of their resultant. *Answer.* $\sqrt{8}$lbs., bisecting angle between directions of 3 and 4.

7. Show that three forces, acting at a point, and represented by 3, 5, 9, cannot, under any circumstances, keep it at rest.

8. Draw two lines, A B, A C, at right angles to each other. Suppose that a force of 10lbs. acts on the point A from A to B, and a force of 12lbs. on the point A from A to C: find, by a construction made to scale, the magnitude and direction of the force which acting at A would balance these forces.

9. A thread A B is stretched by two unequal forces, one of 5lbs. acting from A to B, and one of 3lbs. acting from B to A. (*a*) What is the magnitude and direction of their resultant? (*b*) What is the magnitude and direction of the force that balances them? *Answer.* (*a*) 2lbs. from A to B; (*b*) 2lbs. from B to A.

10. Three forces of 49, 50, and 51lbs. act at a point at angles of 120°: find the magnitude of the resultant, and its inclination to the force of 50lbs. *Answer.* $\sqrt{3}$, at right angles to the force of 50lbs.

11. Find the resultant of two forces of 8lbs. and 10lbs. which are inclined to each other at an angle of 30°. *Answer.* 17·394lbs.

12. If three equal forces acting on a body keep it at rest, how must they be inclined to each other?

13. Draw an equilateral triangle A B C. Suppose a force of 10lbs. to act from B to C, another of 10lbs. from A to B, and a third force of 15lbs. from A to C: find their resultant, and show by a diagram exactly how it acts. *Answer.* 25lbs. parallel to A C.

14. Find the resultant of two forces of 10lbs. and 20lbs. acting at an angle of 60°. *Answer.* 26·45lbs.

15. If a body be held in equilibrium by three forces acting in one plane, show that their lines of direction must either be parallel or pass through the same point.

[If they are not all parallel some two will meet in a point. They may be replaced by their resultant passing through that point. Now this resultant is balanced by the third force, which must, therefore, be equal and opposite to it, and consequently must pass through the same point.]

16. The magnitudes of three forces acting on a particle, and keeping it at rest, are as the numbers 4, 5, 6 respectively, and the position of the line of action of the least force is given: show by a figure the lines of action of the other two.

[Draw a triangle, B D F (Fig. 6), whose sides are 4, 5, 6, and mark an arrow on each side in the same direction round the triangle. Then from any point draw lines parallel to these sides, in the same direction, and of the same length; these will represent the mode of action of the forces.]

17. Forces represented by 3, 4, and 5lbs. act on a particle and keep it at rest: show how to find the angles at which the forces are inclined to each other.

18. Three forces of 8, 10, and 12 units respectively keep a particle in equilibrium: determine by construction how they act.

*19. Three forces act perpendicularly at the middle points of the sides of a triangle, and each force is proportional to the side on which it acts: show that they equilibrate.

*20. A cord whose length is $2l$ is fastened by its ends at A and B, in the same horizontal line, the distance of which is $2a$, a weight W is sustained on the cord by a smooth ring: find the tension.

Answer. $T = \dfrac{Wl}{2\sqrt{l^2 - a^2}}$

*21. Forces of 30, 40, and 50lbs. act on a particle A in the same plane in directions inclined respectively 30°, 45°, and 60°, all on the same side to a straight line A x drawn through A: find the resultant. Answer. 117·38lbs.

22. Resolve a force of 100lbs. into two components, each acting at an angle of 45° with the original force. *Answer.* Each component = 70·71lbs.

23. A force of 3 tons is exerted along a line inclined to the horizontal at an angle of 30°; resolve it into horizontal and vertical components. *Answer.* 2·6 and 1·5 tons.

24. Resolve a force of 100lbs. into two components, one making an angle of 30°, the other of 60°, with the given force. *Answer.* 86·6 and 50lbs.

*25. A particle is kept at rest by three forces (1) 30lbs., (2) 40lbs. at right angles to (1), and a third force inclined to (2) at an angle

whose sin. is $\frac{3}{5}$: find that force. [The line of action of the third force, when produced, falls between the lines of action of (1) and (2).] *Answer*. 50lbs.

SECTION II.

PARALLEL FORCES.

73. **Parallel Forces.**—If two equal parallel forces act on a body in the same direction, they tend to move it in their own direction. Their resultant will evidently be at the point midway between the points of application. Thus, if two equal weights, P and Q, are suspended from the ends of the rod A B, the rod must be supported at its middle point C (Fig. 16).

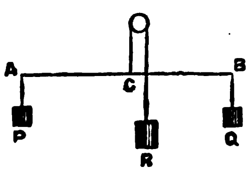


FIG. 16.

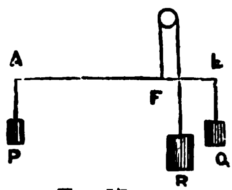


FIG. 17.

74. If two unequal weights, P and Q, be attached to the ends of the rod A B (Fig. 17), the rod must be supported at a certain point, to remain horizontal. But that point will not now be the centre. Making the experiment, this point, F, is found to be always nearer the greater weight. The two parts, A F, F B, have the same ratio to each other as the weights, Q, P. This is expressed by saying that the arms are *inversely* proportional to the weights. The longer arm bears the smaller weight, the shorter arm the greater weight. Moreover, the force R, required to balance P and Q, must be equal to their resultant, and opposite to it. Now this is always found to be equal to the sum of P and Q (neglecting the weight of the rod A B). Hence:—

The resultant of two concurrent parallel forces is parallel to each force, and equal to their sum; and is applied at a point which divides the line joining their points of application into parts inversely proportional to the forces.

*75. This proposition is proved by the composition and resolution of forces meeting in a point. Let A G and B H

(Fig. 18) represent two unequal parallel forces P and Q. Join A B; at A and B introduce two opposite and equal forces, A C, B D: these will not affect the forces P and Q. The resultant of A G and A C is A E; that of B H and B D is B F. Transfer these resultants back along their lines of direction, which can easily be shown to meet, to O, and resolve them in directions parallel to the given forces and to A B. We thus get O C′ and O D′, which are equal and opposite, and may be neglected; and two forces acting along O K towards K, one equal to A G, the other to B H. Let O K cut A B in M, and make M K=A G+B H: M K represents the resultant. Then in the similar triangles E G A and A M O, we have E G : G A :: A M : M O ∴ E G. M O=A G. A M. And in the similar triangles F H B and B M O.

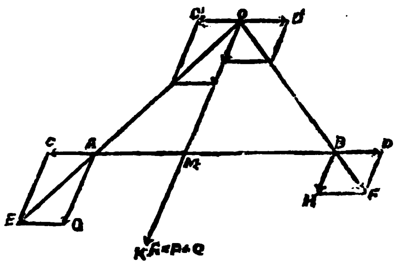


FIG. 18.

F H : H B :: B M : M O ∴ F H. M O=B H. B M.

But E G=F H and ∴ E G. M O=F H. M O.

∴ A G. A M=B H. B M : or $P \times$ *its arm* $= Q \times$ *its arm.*

Hence A G : B H :: B M : A M; or, *the arms are inversely proportional to the forces.* From this proportion we have A G + B H : B H :: B M+A M : A M; or, P+Q : Q :: A B : A M ∴ $AM = \frac{Q \times AB}{P+Q}$

Hence, to find A M, multiply the less force by the distance between the forces, and divide the product by the sum of the forces.

*76. *The resultant of two non-concurrent parallel forces is equal to their difference, in the direction of the greater, and applied at a point found by producing the line joining their points of application beyond that of the greater, so that the greater force may be to the less as the whole produced line is to the produced part.*

Let A G and B H (Fig. 19) be two unlike parallel forces acting at A and B: it is required to find their resultant. At

A and B introduce two equal opposite forces, A C and B D. The resultant of A G and A C is A E; that of B H and B D, B F. Transfer these resultants along their lines of direction to meet in O. At O resolve each resultant in directions parallel to the given forces and to A B. We thus get O C′ and O D′, which are equal and opposite, and therefore may be neglected; and the two opposite forces O G′ and O H′, which are equal to the original forces. The resultant R therefore is the difference of A G and B H in the direction of the greater, and acting along the line M G′, cutting B A produced, in M. Make M K = A G − B H, in the same direction as A G; M K represents the resultant. Then in the similar triangles E G A and A M O, we have E G : G A :: A M : M O ∴ E G.M O = A G.A M. And in the similar triangles F H B and B M O, we have F H : H B :: B M : M O ∴ F H.MO = B H.BM.

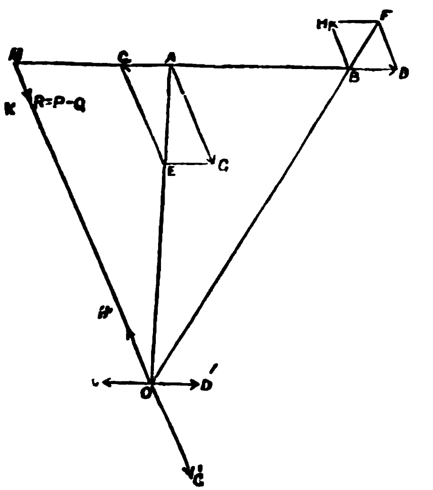


FIG. 19.

But E G = F H ∴ E G.M O = F H.M O.

∴ A G.A M = B H.B M; or, $P \times$ *its arm* $= Q \times$ *its arm.* Hence A G : B H :: B M : A M; or, *the arms are inversely proportional to the forces.* From this proportion we get A G − B H : B H :: B M − A M : A M; or, P − Q : Q :: A B : A M

$$\therefore \text{A M} = \frac{\text{Q} \times \text{A B}}{\text{P} - \text{Q}}$$

Hence, to find A M, multiply the less force by the distance between the forces, and divide the product by the difference of the forces.

It may be observed that when three parallel forces P, Q, and P+Q, balance, the greatest, P+Q, is always between the others and opposite in direction to them both.

77. **Resolution of Parallel Forces.**—*To resolve a given force* R *into two parallel forces acting at given distances on each side of it;* say, as the sum of the distances is to the greater distance, so is R to the greater component; subtract this from R for the less component. *To resolve a given force* R *into two parallel forces acting at given distances on the same side of it.* As the difference of the arms : the shorter arm :: R : the smaller component; add this to R for the greater component.

78. **A Statical Couple.**—If two non-concurrent parallel forces differ very little, we see that their resultant becomes very small, but its point of application is more remote. Now if the two forces be equal, the resultant will be nothing, and its point of application will be removed beyond any assignable distance. Here, in fact, we get $R=0$, and $AM=\infty$. The only effect of such a combination is a tendency to cause the body to rotate round an axis. Such a combination, that is, two equal parallel forces, acting in opposite directions, is called a *couple* or *pair*. Its value or *moment* is expressed by the product of either of the forces by the length of the perpendicular distance between them (Fig. 20). If the moments of the couples in the figure be equal, the couples are said to be equal. We may now make the following statement: When two forces act in a plane, either they balance, so that their resultant$=0$, or they have a single resultant, or they form a couple.

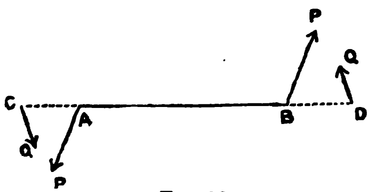


FIG. 20.

79. **The Moment of a Force,** with respect to any point, is the product of the force into the perpendicular from the point on the direction of the force. Thus the moment of the force B C about the point O is equal to $BC \times OP$ (Fig. 21). Moment is thus measured by the product of the number of units of force by the number

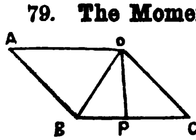


FIG. 21.

of units of length in the perpendicular drawn upon its line of action from the point about which the moments are taken. We thus obtain a number which is proportional to the turning power of the force, and equal to the numerical value of that turning power, if the unit of turning power be taken as the turning power of unit force at unit distance.

The usual arrangement of signs for moments results from that for forces and for the straight lines representing them. Thus, lines drawn to right and upwards in a diagram would represent a turning power in a direction opposite to that of the hands of a watch; these lines are positive, therefore the moment of a force is *positive* when it tends to produce rotation in the direction opposite to that of the hands of a watch; *negative*, when it tends to produce rotation in the same direction as that of the hands of a watch. For example, the moment of O A about C (Fig. 22) is positive; that of O B is negative. Either may be represented by the area of the parallelogram A O B C, or by twice the area of the triangle O B C.

*80. **Principle of Moments.**—(A) *Direct.* If two forces meet in a point, their moments about any point in their resultant are equal. Let O A, O B (Fig. 22), represent the forces P and Q, and O C the resultant. From C, any point in O C, draw C E, C F, perpendicular to O A, O B, produced. Then since A O B C is a parallelogram, the two triangles O B C and O A C are equal. But the triangle O B C is equal to $\frac{\mathrm{O\,B} \times q}{2} = \frac{\mathrm{Q}q}{2}$; and O A C is equal

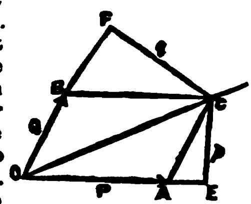


FIG. 22.

to $\frac{\mathrm{O\,A} \times p}{2} = \frac{\mathrm{P}p}{2}$. Therefore $\mathrm{Q}\,q = \mathrm{P}\,p$; or, the moments of P and Q about C are equal. But their moments are different in kind, that of P being positive, that of Q negative. (B) *Inverse.* If the moments of two forces meeting in a point be equal and opposite with respect to any point lying in their plane, that point is on their resultant. Let the moments of P and Q about C be equal and opposite, we have to prove that C is a point on the line of action of the resultant. $p\,\mathrm{P} = q\,\mathrm{Q} \therefore p : q :: \mathrm{Q} : \mathrm{P}$. But $p : q :: \mathrm{C\,A} : \mathrm{C\,B}$. Therefore $\mathrm{C\,A} : \mathrm{C\,B} :: \mathrm{Q} : \mathrm{P}$, or $\mathrm{B\,O} : \mathrm{O\,A} :: \mathrm{Q} : \mathrm{P}$. The two sides B O, O A have the same ratio as the forces. They may

therefore be taken to represent them. Therefore the diagonal O C will represent the resultant, in direction.

(C) The algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point.

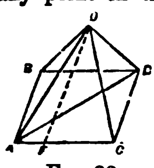


FIG. 23.

Let A B, A C represent the forces, and A D the resultant, and let O be a point in their plane. Then A O D=A O D C−A D C=A O D C−A B O−O D C=A O C−A B O. ∴2 A O D=2 A O C−2 A B O; *i.e.*, moment of A D about O= moment of A C−moment of A B. But the difference (moment of A C−moment of A B) is the *algebraic sum* of the moments of A B and A C in this case, because they turn opposite ways about O. A like result may be proved for all positions of O in the plane, whether inside the parallelogram or outside, within the angle B A C or not.

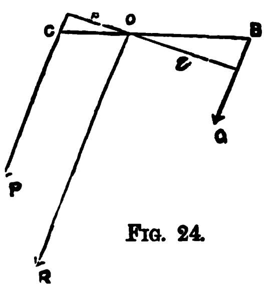


FIG. 24.

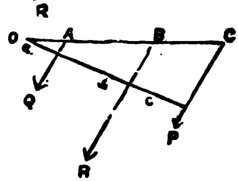


FIG. 24A.

*81. **Moments of Parallel Forces.**—(1) The moments of P and Q with respect to O, a point on their resultant, are equal (Fig. 24). For (by Art. 74) P : Q :: O B : O C, and O B : O C :: q : p. Therefore P : Q :: q : p ∴ pP=qQ. (2) If the moments of P and Q about O be equal, O is on the resultant. For pP = qQ ∴ p : q :: Q : P, that is, the perpendicular, and therefore also, B C, the line between the points of application, is divided at O inversely proportional to the forces; therefore O is on the resultant. (3) The algebraic sum of the moments of two parallel forces about any point in their plane is equal to the moment of their resultant about the same point (Fig. 24A).

$R=P+Q \therefore R\times(a+b)=Pa+Qa+Pb+Qb.$
But $Qb=Pc.$
$\therefore R\times(a+b)=Pa+Pb+Pc+Qa.$
$R\ (a+b)=P\ (a+b+c)+Qa$, or $Rr=Pp+Qq.$

If any straight line be drawn across the lines of direction of a series of parallel forces in the same plane,

$$R = P + Q + S + \&c.$$
$$Rr = Pp + Qq + Ss + \&c.$$

p being the distance from any fixed point in the cutting line to its intersection with P, &c.; and R being the resultant; and these being the *algebraic sums* of the forces and moments respectively, subject to the previously agreed convention as to sign.

*82. **Centre of Parallel Forces.**—If any number of parallel forces act on a body, the resultant can be found by repeating the process given above. And if the direction of the forces, but not their magnitudes or points of application, be changed, the resultant will always pass through the same point. Thus, if P Q S (Fig. 25) be the forces, whether they act in the direction of the dark or in that of the dotted lines, the points of application being fixed, the resultant will always pass through the point C. This point, which is called the centre of parallel forces, may be defined *the point through which the resultant of a system of parallel forces always passes,* so long as the magnitudes and points of application of the forces remain the same.

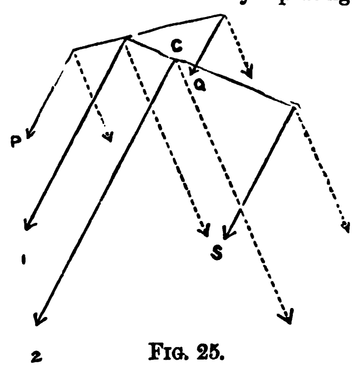


FIG. 25.

EXERCISES.

1. Two like parallel forces of 30lbs. and 40lbs. act at the ends of a rod 7 feet long: by what point must the rod be suspended to remain in equilibrium? *Answer.* 3 feet from the greater force.

2. Weights of 2 tons and 3 tons act at the two points of trisection of a horizontal beam supported at both ends: required the pressure on each point of support. *Answer.* $2\frac{2}{3}$ and $2\frac{1}{3}$ tons.

3. Two non-concurrent parallel forces, of 70 and 30lbs. respectively, act at a distance of 3 feet asunder: find the resultant, and the distance of its point of application from that of the greater force. *Answer.* 40lbs.; 27 inches.

4. Forces of 100 and 150lbs. act in opposite directions, at a distance of 24 inches: find the resultant, and the distance of the point at which it acts from the greater force. *Answer.* 50lbs.; 48 inches.

5. Resolve a force of 20lbs. into parallel forces acting at distances of 10 and 12 inches on different sides of it. *Answer.* $10\frac{10}{11}$ and $9\frac{1}{11}$lbs.

6. Resolve a force of 20lbs. into parallel forces acting at distances of 5 and 12 inches on the same side of it. *Answer.* $34\frac{2}{7}$ and $14\frac{2}{7}$lbs.

7. Like parallel forces of 12, 4, 6, and 8lbs. act in order at equal distances asunder along a straight line: find their centre. *Answer.* $\frac{4}{9}$ of the distance between the outside forces from force of 12lbs.

8. A carriage wheel of weight W, radius r, rests on a horizontal road; find the force required to draw it over an obstacle of height h.

Answer. $F. = \frac{\sqrt{h(2r-h)}}{r-h}.$

CHAPTER IV.

CENTRE OF GRAVITY.

83. **Gravity,** or Gravitation, is the mutual attraction which exists between all bodies. It acts at all distances. It causes bodies to fall to the earth, the earth and other planets to continue connected with the sun; and connects the solar system with other parts of the universe. Applied to the earth, it means the force with which the earth draws all bodies within its influence towards itself. Every body near the earth's surface, if free to move, falls to the ground. Smoke, balloons, &c., seem to be exceptions, but are not really so. They rise because they are displaced by air, which is more strongly attracted by the earth. Every body also attracts the earth towards itself; but terrestrial bodies are so small, comparatively, that their attraction causes no perceptible motion in the earth. If a body of the same size and material as the earth were placed near it, such body and the earth would both move and they would meet half-way.

84. **Direction of Gravity.**—Consider any *particle* outside the earth. It is attracted by every particle of the earth in its separate direction; the resultant of all these attracting forces is a force which must pass through the particle itself and act in some straight line towards the earth. By suspending the particle with a string (as in a plumb line) we can find out the direction of the earth's attraction upon it. It can be proved that if the earth were a perfect sphere, and of the same material throughout, its attraction on such an external particle would in all cases be towards its geometrical centre. As these two conditions are not perfectly fulfilled, we cannot say that all such forces pass exactly through the centre, but experiment shows that they must at least pass very near it. We shall therefore speak of the geometrical centre of the earth considered as a sphere as the centre of the earth's attraction.

Next consider a *body* outside the earth. Every particle of this body is attracted by a force towards the earth's centre $\therefore$ the resultant attraction of the earth on the body also passes through the centre. The attractions on two particles or two bodies near one another are not parallel forces, because they meet at the centre; but as their lines of action begin near together, and yet do not meet for nearly 4,000 miles, they may be treated as parallel without introducing any appreciable error.

85. The total force acting on any body to cause its fall to the earth is the resultant of the forces acting on all its particles; and, since these are parallel, the resultant is their sum. This is called the *weight* of the body. Its point of application, **The Centre of Gravity**, may be thus defined :—

(1) *The centre of gravity of a body is the point at which the resultant attraction of the earth upon it, regarded as the resultant of a series of parallel forces, always acts, however much the position of the body is altered, provided that no alteration is made in the shape of the body, or in the relative position of its particles;* or thus :—

(2) *The centre of gravity of a system of heavy particles* (whether forming one body or not) *is a point such that if it be supported, and the particles rigidly connected with it, the system will rest in all positions, provided it is acted on only by the earth's attraction.*

The second is recommended as the more convenient form for a concise definition. A body or system cannot have more than one centre of gravity.

86. Whenever a surface or a body of uniform density contains what is called a *centre of symmetry—i.e.*, a point which bisects all straight lines drawn through it from one part of the surface of the body to another—that point is the centre of gravity. Hence (1) in a straight line or rod the centre of gravity is the middle point; (2) in a circle, or the circumference of a circle, it is the centre; (3) in a parallelogram, the intersection of the diagonals; (4) in a sphere, the centre; (5) in a cylinder, the middle point of the axis; (6) in a cube, the common intersection of its diagonals. The centre of gravity of a body is sometimes not in the body. Thus, the centre of gravity of a ring is its centre.

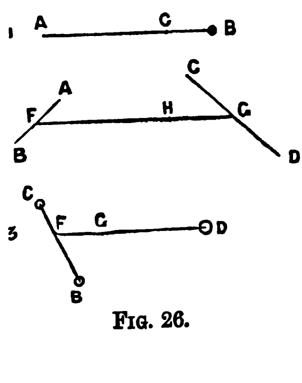


FIG. 26.

87. To find the centre of gravity in any case is the same as to find the centre of a system of parallel forces (82).

(1) *To find the centre of gravity of two heavy particles at A and B, whose weights are P and Q.* Divide A B in C, so that $\frac{BC}{AC} = \frac{P}{Q}$. C is the centre of gravity.

(2) *To find the centre of gravity of two heavy lines (A B, C D).* Bisect the lines in F and G. Join F G. Divide it in H, so that A B : C D :: G H : H F.

(3) *To find the centre of gravity of three bodies, whose centres of gravity are at B, C, D, and whose weights are P, Q, R.* Join B and C, and cut B C in F, so that $\frac{\text{C F}}{\text{B F}} = \frac{\text{P}}{\text{Q}}$. Join F D, and divide F D in G, so that $\frac{\text{D G}}{\text{F G}} = \frac{\text{P} + \text{Q}}{\text{R}}$. G is the centre of gravity of the three bodies.

88. *To find the centre of gravity of a triangle, i.e., of a triangular lamina or plate of uniform and very small thickness.*—The triangle A B C may be considered as made up of a number of material lines laid side by side, parallel to B C. Bisect B C in D, and join A D: A D can easily be shown to bisect all lines drawn across the triangle parallel to B C. The centre of gravity of each line being its middle point, the centre of gravity of the whole triangle will be in the line A D passing through all the middle points. Again, we may consider A B C as made up of material lines parallel to A C. Therefore the centre of gravity of the triangle will be in the line B F, passing through the points of bisection of these lines. The centre of gravity is therefore G, where these lines intersect.

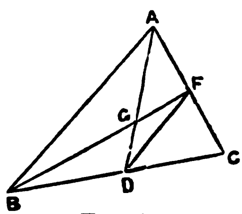


FIG. 27.

The position of this point can readily be determined. Join F D. Then it is easily shown by geometry that F D is parallel to A B and half of it. Therefore the triangle D F G is similar to A B G, and hence D F : A B :: D G : G A. That is, since D F is half of A B, D G is half of A G, or D G is one-third of A D. So also F G can be shown to be one-third of F B. It is easily proved that the centre of gravity of a triangle is the same as that of three equal weights placed at its angles. For, let W be the weight at each angular point, then the resultant of W at B and W at C is 2 W at D, the middle point of B C. The resultant of 2 W at D and W at A is 3 W at a distance x from D, so that $2\text{ W}x = \text{W}(\text{A D} - x)$ $\therefore\ 2x = \text{A D} - x\ \therefore\ 3x = \text{A D}$, and $x = \frac{1}{3}\text{ A D} = \text{D G}$ $\therefore$ the centre of gravity of the three equal weights is at G, which is also the centre of gravity of the triangle.

*89. *To find the centre of gravity of a trapezoid.*—Let A B D C be a trapezoid, of which A B, C D, are the parallel sides. Join B C. Bisect A B in E, and C D in F; join C E, B F, and E F. Take E H $=\frac{1}{3}$ E C, and F K $=\frac{1}{3}$ F B $\therefore$ H is the centre of gravity of the triangle A B C, and K of B C D. The centre of gravity of the trapezoid lies in H K, the line joining these points; but it also lies in E F, the line joining the middle points of the parallel sides. Therefore the centre of gravity is G, the point where the lines H K, E F, intersect each other. Draw H L, K M, parallel to A B or C D, and meeting E F in L and M. Let A B be called b, and C D a; also let G F be called x, and G E y. Since the triangles F M K, F E B, are similar, M K : E B :: F K : F B $\therefore$ M K $=\frac{1}{3}$ E B $=\frac{1}{3}b$. So also the triangles H L G, K M G, are similar $\therefore$ G M : G L :: M K : H L. $\therefore$ G F $-$ F M : G E $-$ E L :: M K : H L. But E L $=$ F M $=\frac{1}{3}$ E F $=\frac{1}{3}(x+y)$.

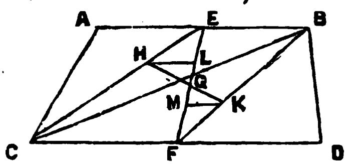


Fig. 28.

$\therefore\ x-\frac{1}{3}(x+y) : y-\frac{1}{3}(x+y) :: \frac{1}{3}b : \frac{1}{3}a.$

$\therefore\ 3x-x-y : 3y-x-y :: b : a.$

$\therefore\ 2x-y : 2y-x :: b : a.$

But $y=$ E F $-x$. Substituting this, we get $3x-$ E F . 2 E F $-3x$ :: $b : a$, or $a\,(3x-\text{E F}) = b\,(2\,\text{E F}-3x)$.

$\therefore\ 3ax - a\,\text{E F} = 2b\,.\,\text{E F} - 3bx.$

$\therefore\ 3x\,(a+b) = \text{E F}\,(a+2b)$

$$\therefore\ x = \frac{\text{E F}}{3} \times \frac{a+2b}{a+b} = \frac{\text{E F}}{3} \times \frac{\text{C D}+2\,\text{A B}}{\text{C D}+\text{A B}}.$$

90. *To find the centre of gravity of any rectilineal figure.*—Divide the figure into triangles, by lines drawn from one angular point, and find the centre of gravity of each. Join the centres of gravity of two triangles, and divide the joining lines into parts inversely proportional to the areas of the triangles. This will give the centre of gravity of the quadrilateral made up of the two triangles. Join this with the centre of gravity of a third triangle, and divide the joining line into parts inversely proportional to the quadrilateral and the third triangle. The point so found will be the centre of gravity of the figure made up of three triangles. By continuing the process the centre of gravity of the whole figure may be found.

*91. *To find the centre of gravity of a triangular pyramid.* Let A B C D be the pyramid. Bisect the edge B D in E, join C E, and take E F=$\frac{1}{3}$ C E ; join also A E, and make E H=$\frac{1}{3}$ A E. Join A F, C H. The pyramid may be supposed to consist of thin triangular plates parallel to A B D ; the centre of gravity of each of these would lie in the line C H $\therefore$ the centre of gravity of the pyramid lies in C H. The centre of gravity of the pyramid lies also in A F for a similar reason. Therefore the centre of gravity lies in G, the point in which A F, C H intersect. Because E F = $\frac{1}{3}$ C E, and E H = $\frac{1}{3}$ A E $\therefore$ F H is parallel to A C and one-third of it. The triangles A C G, G F H, are similar ; therefore H G : G C :: H F : A C $\therefore$ H G=$\frac{1}{3}$ G C, or H G=$\frac{1}{4}$ C H. Also F G=$\frac{1}{4}$ A F. Hence to find the centre of gravity of a triangular pyramid, join any apex with the centre of gravity of the opposite surface, and divide the joining line so that the segment next to the surface may be $\frac{1}{4}$ of the whole line. The centre of gravity of a triangular pyramid coincides with that of four equal weights placed at its angles.

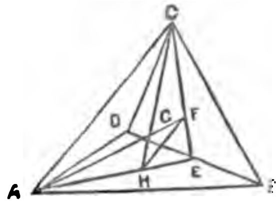


FIG 29.

To find the centre of gravity of any pyramid.—Find the centre of gravity of the base, and join it with the apex. Divide the joining line so that the segment towards the base is one-fourth of the whole line. This follows from the last demonstration. *A cone* may be regarded as a pyramid with an infinite number of sides, and its centre of gravity is found in the same way. We can practically find the centre of gravity of any body in an easy manner, especially if the body be thin. Suspend the body from one point, and mark a vertical line through that point; then suspend it from a different point, and make a vertical line through that point. The centre of gravity is in each of these verticals, and is therefore in the point where they intersect. Every schoolboy has thus discovered the centre of gravity of his slate.

92. *Given the centre of gravity of a body and the centre of gravity of part of it, to find the centre of gravity of the remainder.*

Let W be the weight of the whole body and G its centre of gravity; let W_1 be the weight of the given part, and G_1 its centre of gravity; then $W - W_1$ is the weight of the remainder, and let G_2, its centre of gravity, be the required point:—

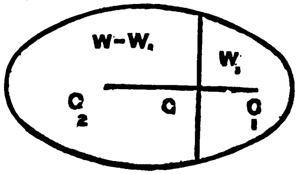


FIG. 30.

$$W_1 : W - W_1 :: G_2G : G_1G$$

$$\therefore GG_2 = \frac{W \times GG_1}{W - W_1}$$

Multiply the weight of the given part by the distance of its centre of gravity from the centre of gravity of the whole body, and divide by the weight of the remainder. This gives the distance of the centre of gravity of the required part from the centre of gravity of the whole.

93. **Equilibrium of a body capable of turning about a fixed point or fulcrum.**—To preserve equilibrium the centre of gravity must be supported (85). This will be done if the centre of gravity is in the same vertical line with the fixed point, either above, below, or coinciding with it.

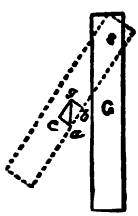


FIG. 31.

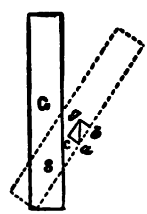


FIG. 32.

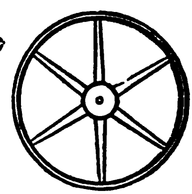


FIG. 33.

(1.) If S, the point of suspension, be above G (Fig. 31), the centre of gravity, and the body be moved to one side, the effect of gravity will be to pull the body back to its position, so that G shall be again vertically below S. The equilibrium is in this case called *stable*. G *a*, representing gravity, acts vertically downward; it may be resolved into G *c*, which simply produces a strain at S, and G *b* which moves the body to its original position, so that G is again vertically below S.

(2.) If the point of suspension, S, be below the centre of gravity (Fig. 32), and the body be displaced, the effect of gravity will be to make the body fall farther and farther away

from its position, so that G, moving round S, shall come to rest in a position beneath it. This is *unstable* equilibrium.

(3). If the point of suspension coincides with the centre of gravity (Fig. 33), the body will remain at rest in any position whatever. For example, when a wheel is suspended by its centre it remains at rest indifferently in all positions. Such equilibrium is called *neutral*, or *indifferent*.

94. **Equilibrium of a body resting on a horizontal plane.**—If a body rests on a plane, it must touch it either in one point or in more than one. If it touch the plane in only one point, the centre of gravity and the point of contact must, to preserve equilibrium, be in the same vertical line. The equilibrium will be *stable* when the centre of gravity occupies the lowest position possible. Thus, if a globe of wood have a quantity of lead run into one part of its surface, the equilibrium will be stable when that portion is in contact with the plane. If the centre of gravity occupies any other position than the lowest the equilibrium will be *unstable*; and in the case of a globe of uniform density the equilibrium will be *neutral*.

95. If the body touch the plane at several points there will be equilibrium, if the vertical through the centre of gravity falls within the figure formed by joining the points of contact, without any re-entrant angles. The nearer it is to any side of this figure the more easily will the body be overturned on that side. If a round table be supported by three equidistant legs, A, B, C (Fig. 34), the centre of gravity, being in the middle point, will be supported, and the table will have no greater tendency to fall over at one side than at the other. If, however, a heavy weight be placed at D, it may overturn the table at that side by moving the centre of gravity outside the line A B. If the vertical through the centre of gravity falls outside the base, the body will fall. The higher the centre of gravity is for the same base, the more easily is the body overturned, hence the danger of placing too much luggage on the top of a carriage, or of passengers standing up in a small boat; also of deck-loading in ships.

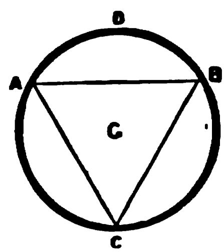


FIG. 34.

EXERCISES.

1. Find the centre of gravity of 20lbs. and 36lbs. at a distance of 12 inches. *Answer.* $4\frac{2}{7}$ inches from the larger weight.

2. Weights of 3, 4, and 5lbs. are placed at the angular points of an equilateral triangle whose side is 12 inches; find their centre of gravity.

3. Find the centre of gravity of a board of a polygonal figure.

4. Find the centre of gravity of a body in the shape of a T square.

5. Draw a triangle, A B C, and suppose it to represent a triangular board weighing 10lbs. Suppose weights of 5lbs., 5lbs., 10lbs, are placed at A, B, C, respectively; where will the centre of gravity of the whole mass now be? *Answer.* $\frac{4}{9}$ distance along line drawn from middle point of A B to C.

6. Mention an experimental way of showing that the centre of gravity of a circular board is at its centre.

7. Find the centre of gravity of a solid formed of two unequal spheres touching each other. The radius of one is 4 inches, weight 10lbs.; the other, whose radius is 10 inches, weighs 20lbs. *Answer.* $4\frac{2}{3}$ inches from centre of larger.

8. Find the centre of gravity of the portion remaining when from a square has been cut away a portion by a line drawn from one angle to the middle of one of the opposite sides.

9. Find the centre of gravity of a trapezium.

10. Show that a system of bodies can have only one centre of gravity.

11. An equilateral triangle, whose perpendicular is 12 inches, is divided into two parts by a line parallel to one side, through the centre of gravity of the triangle; find the centre of gravity of the quadrilateral portion of it. *Answer.* $1\frac{13}{15}$ inch from middle point of greater side.

12. A bar of iron 15 inches long, of uniform thickness, weighing 12lbs., has a weight of 10lbs. suspended from one extremity; where must a prop be placed so that the bar will just balance on it? *Answer.* $3\frac{9}{22}$ inches from the middle point.

13. A bar of uniform thickness weighs 10lbs. and is 7 feet long; weights of 6lbs. and 13lbs. are suspended from its extremities; where will it balance? *Answer.* $2\frac{19}{29}$ feet from the greater weight.

14. If a triangular slab be supported on three props at its angular points the pressures on the props are equal, whatever may be the weight of the triangle.

15. A circular table resting on three legs under its edge, and equally distant from each other, weighs 60lbs.: what is the least weight which can be placed on the table so as to overturn it? *Answer.* 60lbs.

16. Find the centre of gravity of three equal weights, placed so s to form a triangle.

17. Two balls of uniform density are placed side by side in contact; the one weighs 120lbs., the other 360lbs. Find how far the centre of gravity of the two balls is from the centre of gravity of the heavier. *Answer*. $\frac{1}{4}$ of the sum of the radii.

18. A square board weighs 4lbs., and a weight of 2lbs. is placed at one of its corners. Show by a figure the position of the centre of gravity of the board and the weight. *Answer*. $\frac{1}{6}$ of the diagonal from centre.

19. What is the position of the centre of gravity of that portion of a cube which remains when one corner has been cut off by a plane which passes through three adjacent corners? *Answer*. ·057165 of length of edge from centre on side remote from part cut off.

20. A rectangular slab (specific gravity $=8$), 1 inch thick, 10 inches long, 8 inches broad, is placed on a table, with one side (whose edge is parallel to the edge of the table) projecting beyond it. A weight of 1lb. is placed on the opposite edge: how far can the slab be pushed off the table without falling? *Answer*. $5\frac{175}{358}$ inches.

21. A cube is placed on a rough inclined plane: through what angle can the plane be raised without causing the cube to topple over? *Answer*. 45°.

22. A cone, radius of base $=2$ inches, height $=8$ inches, is placed on an inclined plane; determine the greatest inclination the plane can have before the cone topples over, supposing it to be prevented from sliding. *Answer*. 45°.

23. Find the centre of gravity of half a regular hexagon.

CHAPTER V.—MACHINES.

THE MECHANICAL POWERS.

96. **A Machine** is an instrument by which one force is made to resist or overcome another force not directly opposite to it in direction. In every machine, therefore, there are two forces to be considered, viz., *the power*, or force applied to the machine, and *the weight*, or resistance, which the power is employed to overcome.

97. Two forces will balance each other if they are equal and opposite, but not otherwise. Hence, P, the power, should be equal to R, the resistance, if the machine were free, *i.e.*, not in contact with fixed points, lines, or surfaces. But all machines contain certain fixed points, lines, or surfaces, and these resist pressure, and therefore support part of the weight.

98. The resistance which a fixed point, line, or surface offers to pressure, or any other force, is called its *reaction*. Fixed points react *in all directions*; fixed lines and surfaces (smooth) only in directions at *right angles to themselves*, *i.e.*, in the direction of a normal or perpendicular to themselves.

99. **General Principle of Equilibrium of Machines.**—If P and R be the forces applied to a machine, *the machine will be in equilibrium when the resultant of P and R passes through a fixed point, or is perpendicular to a fixed line or surface of the system.* If the resultant does not pass through a fixed point, line, or surface, it is unresisted, and must produce motion. In every machine the *fulcrum* or fixed point, &c., bears a strain or pressure equal to this resultant.

100. *Simple Machines or Mechanical Powers* are the simplest forms of mechanical appliances, and may be considered as the elements from which all machines are formed. They are six in number:—Lever, Wheel and Axle, Inclined Plane, Wedge, Screw, and Pulley.

THE LEVER.

101. **Levers.**—A lever is an inflexible rod, capable of turning freely about a fixed point. There will be equilibrium when the resultant of all the forces acting on the lever passes through the fixed point. The lever is usually applied to resist or overcome a force applied at one point of it

by means of a force applied at some other point. The former is called the *resistance*, or weight; the latter, the *power*. The fixed point is the *fulcrum*, and the portions of the lever between the fulcrum and the points where the forces are applied are called the *arms*.

102. The effect of a lever varies according to the position of the fulcrum, power, and resistance. Hence there are three *orders* of levers, as shown in Fig. 35.

The **first order** has the *fulcrum intermediate* between the power and resistance. Thus, in stirring a fire, if I rest the poker on the bar of the grate, the bar is the fulcrum; I apply the power at one end, and the weight of the coal at the other end is the resistance. Here the power and weight act in the same direction, and either may be the greater, or both may be equal.

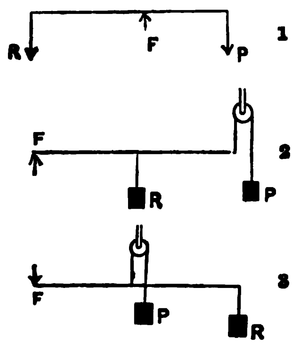


FIG. 35.

The **second order** has the *resistance intermediate*. The power and weight act in opposite directions, and the weight is the greater. A wheel-barrow may be taken as an example: the fulcrum is at the axle, the power at the ends of the shafts, and the weight intermediate, in the barrow. In the oar of a boat, the resistance to be overcome is the weight of the boat, the fulcrum is where the oar dips into the water, and the power is applied by the hands at the handle of the oar.

In the **third order** the *power is intermediate*, is greater than the weight, and acts in an opposite direction. The fore-arm is an example. The fulcrum is at the elbow, the weight in the hand, and the power is applied at a point between, by the contraction of the muscle. The treadle of a lathe or of a spinning wheel is another instance of this kind of lever.

The door of a room is a lever of the second or third order, according to the point at which the power is applied. The resistance is at the centre of gravity of the door; if we apply the power further from the hinge than that point, the door is a lever of the second order, if nearer to the hinge, it belongs to the third order.

Double Levers are exemplified in scissors and pincers, whic'

are of the first order, nutcrackers of the second order, and tongs of the third.

103. If the power and weight be parallel to each other and perpendicular to the arms, their relative magnitude depends on the length of the arms which carry them. They are then inversely proportional to those arms, that is—

P . R : : arm bearing R : arm bearing P, or

P $\times$ its arm = R $\times$ its arm; or $P p = R r$. That is, the moments of P and Q, with respect to the fulcrum, are equal.

This must be the case in order that the resultant should pass through the fulcrum.

But if the moments of the forces which tend to turn the lever in one direction exceed the moments of those which tend to turn it in the opposite direction, it must move in the former direction.

104. The *mechanical advantage* of the lever is the ratio of the weight to the power. Thus, if a power of 10lbs. balance a weight of 50lbs., the mechanical advantage is 5. In levers of the first order mechanical advantage is gained or lost according to the position of the fulcrum. If P has the longer arm, R must be greater than P; if the shorter, R is less than P. In the second order mechanical advantage is always gained; in the third order always lost. Where there is a loss in power, however, there is always gain in velocity.

105. *When the weight of the lever is taken into account*, this weight, w, may be considered as a power acting at the centre of gravity of the lever. Let the centre of gravity of the lever be at a distance, d, from the fulcrum. (1) If w acts with the power (Fig. 36) W must balance both P and w. Therefore $W b = P a + w d$. (2) If w assists the weight (Fig. 37) P must balance both W and w. Therefore $P a = W b + w d$; *or, the moments tending to turn the lever in one direction are equal to those tending to turn it in the opposite.*

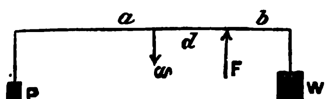


FIG. 36.

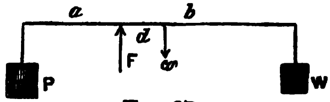


FIG. 37.

If the lever be uniform, we may simply add half its weight to the power and half to the weight, and determine the arms by the method already given.

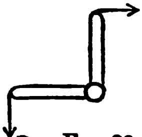


FIG. 38.

106. **The Bent Lever.**—In the bent lever (Fig. 38), the same rules apply. P × its arm = W × its arm. The claw-hammer and the bell-crank are examples. The bent lever is convenient when the power and resistance act in different directions.

107. The foregoing rules apply only when the power and resistance act at right angles to the lever. When this is not the case (as in Fig. 39), P : R : : perpendicular from fulcrum on direction of R : perpendicular on direction of P. Hence, $P\,p = R\,r$, or the moments of P and R about the fulcrum are equal.

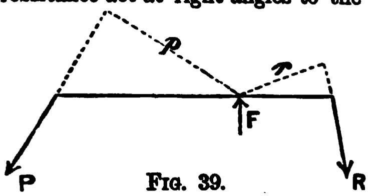


FIG. 39.

108. *Strain on the fulcrum.*—When the power and weight are parallel, and act on a straight lever, the strain on the fulcrum is easily determined. In the first order the strain is equal to the sum of the forces, in the second to the excess of the weight over the power, and in the third to the excess of the power over the weight.

In other cases the strain on the fulcrum is equal to the resultant of the power and weight, and can be determined by the usual rules for the composition of forces.

109. *Combinations of Levers.*—The three orders of levers are often combined in various ways for special purposes. A combination of three levers is represented in Fig. 40. The upper lever is of the first, the next of the second, and the lowest of the third order. The mechanical advantage is found by multiplying W by the arm to which it is attached and by all the alternate arms: P by its arm and all the alternate arms, and dividing the second product by the first.

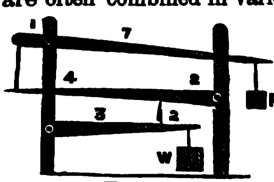


FIG. 40.

$$\frac{W \times 5 \times 2 \times 1}{P \times 7 \times 6 \times 3} = \frac{10\,W}{126\,P} \therefore \text{Efficiency} = \frac{126}{10} = 12{\cdot}6.$$

Examples.

1. In a lever, A C B, 8 feet long, the fulcrum C is 10 inches from B; what weight is required at B to balance 10 lbs. at A?

Here W is 10 lbs. and its arm is 96 − 10 = 86 inches, hence $P \times 10 = 10 \times 86 = 860 \therefore P = \frac{860}{10} = 86$ lbs.

2. In the same lever, suppose the fulcrum at B, what force must be applied at C to balance 10 lbs. at A, and in what direction must it act?

The lever is now of the third order; the weight, 10 lbs., acts now at the whole length of the lever, therefore $P \times 10 = 10 \times 96 = 960 \therefore P = 96$ lbs., and as the weight acts downwards P must act upwards.

3. A rod, A B, weighs 10 lbs., and is found to balance about a point 8 feet distant from A. A weight of 6 lbs. is fastened to A; about what point will the rod now balance?

The whole weight of the rod acts at the point on which it balanced at first. We have therefore only to divide the distance between that point and A, *i.e.*, 8 feet, inversely proportional to the weights.

Thus, as 16 lbs. : 10 lbs. :: 96 inches : 60 inches. It will now balance at a point 5 feet from A.

110. *The Common Balance* is an example of the lever of the first order with equal arms. Since the arms are equal, the forces which balance must also be equal. Hence the balance is used for testing the equality of weights. The Common Balance consists of a beam A B (Fig. 41) suspended from a fulcrum, C, about which it can turn freely. C is a little above the centre of gravity of the beam, G. From the ends of the beam are suspended two scale pans, P, W. W contains the substance whose weight is to be determined, and known weights are placed in P, just sufficient to balance W. When the beam is horizontal, as shown by an index, the weights in P will give the weight of W. To test whether a balance is properly constructed, change the weights into different scales. If the beam still remains horizontal, the balance is true; if not, it is false.

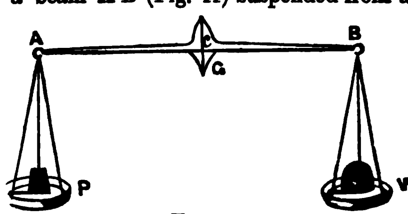


FIG. 41.

111. **Requisites of a good Balance**: (1) When the scales are empty, the beam should be horizontal, and the index vertical. Hence, the centre of gravity of the beam and its

appendages should fall a little below the point on which it is suspended. (2) When equal weights are placed in the scales the index should be vertical, therefore the arms must be of the same length. (3) When the weights are unequal, the balance should readily show this. The readiness with which the index is turned from the vertical, is called the *sensibility* of the balance. It depends on two conditions: (*a*) The arms should be as long as possible; for, the longer the arms, the greater is the moment due to excess of weight. (*b*) The beam should be as light as possible; for, the lighter the beam, the more will it be displaced by an inequality in the weights.

112. *We may weigh with a false balance* in two ways. We may balance the body to be weighed with shot or sand, then remove the body, and put known weights into the scale till equilibrium is restored. These weights will show the *true* weight of the body. Another method is to weigh the body in both scales, multiply the apparent weights together, and extract the square root, which will give the true weight. This may be shown as follows: Let w be the true weight of the body; a its apparent weight in one scale, and b in the other; and x and y the arms. When weighed in the first scale, we have $w\,x = a\,y$; in the second, $w\,y = b\,x$. Therefore $w^2\,x\,y = a\,b\,x\,y$, and $w^2 = a\,b$, or $w = \sqrt{a\,b}$. Thus, if a body weigh 12 lbs. in one scale, and 14 in the other, its true weight is $\sqrt{168}$, or 13 nearly.

113. **The Steelyard** is a kind of lever by which one weight serves to weigh various bodies (Fig. 42). The fulcrum f, and the position of the scale bearing the weight, W, are fixed; the power P is movable. The longer arm is graduated, and the weight of W is shown by the point where P must be placed to balance it.

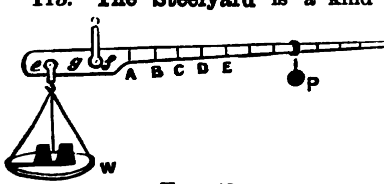


FIG. 42.

114. *To graduate the Steelyard.*—Suppose g to be the centre of gravity of the beam and scale-pan, and let P, when placed at A, balance the instrument. Place 1 lb. in the scale, and let B be the point where P then balances. Let it also balance 2 lbs. at C, 3 lbs. at D, and so on. The portions A B, B C, C D, &c., will all be equal. For since P produces equilibrium at A, if w be the weight of the instrument, $P \times A\,f =$

$w \times gf$ (1). Now P at B balances 1 lb. additional, therefore $P.\,Bf = w.\,gf + 1 \times ef$ (2). Subtract (1) from (2) $\therefore P.\,AB = 1.\,ef$. So P at C balances 2 lbs. additional, therefore $P.\,Cf = w.\,gf + 2\,ef$ (3). Subtract (1) from (3) $\therefore P.\,AC = 2\,ef$. So that B C is equal to A B. The divisions can be shown in a similar way to be equal in the case in which g is to the right of f, and where g and f coincide.

115. **The Danish Balance** (Fig. 43) is a kind of Steelyard, with movable fulcrum. A heavy knob, P, at one end,

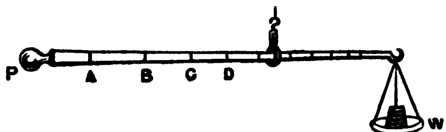


FIG. 43.

serves as a counterpoise to the substance to be weighed. Place 1 lb. in the scale, and let A be the position of the fulcrum which produces equilibrium. Now place 2 lbs. in the scale, and let the fulcrum produce equilibrium at B. The positions C, D, E, &c., can, in like manner, be made to correspond to 3, 4, 5, &c., lbs. The divisions will become smaller and smaller as they approach W.

EXERCISES.

1. A lever, 8 feet long, is used to move a weight of 2 cwt., the power is 20 lbs.; where is the fulcrum? *Answer.* 7·9 inches from weight.

2. What weight will counterbalance a weight of 70 lbs. on a lever 10 feet long, when the fulcrum is 10 inches from the weight. *Answer.* $6\frac{4}{11}$ lbs.

3. A lever, 10 feet long, has a weight of 6 lbs. at one extremity, the fulcrum is 1 inch from the same end; what weight at the other end will the 6 lbs. balance? *Answer.* $\frac{6}{119}$ lbs.

4. If 16 lbs. balance 2 oz. at the end of a lever 8 feet long, where is the fulcrum? *Answer.* $\frac{32}{43}$ inches from the greater weight.

5. A lever, A B, 8 feet long, is supported at A; a weight of 40 lbs. is hung at C, 3 feet from A; what force applied at B will balance it, and what is the pressure on the fulcrum? *Answer.* 15 lbs.; 25 lbs.

6. A rod of uniform section and density weighs 10 lbs.; a weight of 10 lbs. is tied to one end of it, and one of 20 lbs. to the other; under what point of the rod must a fulcrum be placed for the whole to be in equilibrium? *Answer.* $\frac{3}{8}$ of its length from greater weight.

7. A uniform bar weighs 15 lbs. and is 7 feet long; weights of 9 lbs. and 5 lbs. are suspended from its ends; find the point on which it will balance. *Answer.* $3\frac{1}{58}$ feet from the 9 lbs.

8. A bar, 5 feet long and weighing 24 lbs., is supported at its extremities and bears a weight of 1 cwt. at a point 2 feet from one extremity; find the pressure on each support. *Answer.* $79\frac{1}{5}$ lbs. and $56\frac{4}{5}$ lbs.

9. Two weights, P and Q, are placed at the two points of trisection of a horizontal beam supported at both ends; required the pressures they produce at the points of support. *Answer.* $\frac{2}{3}P + \frac{1}{3}Q$, and $\frac{1}{3}P + \frac{2}{3}Q$.

10. A beam, 20 feet long, balances itself on a point $\frac{1}{4}$ of its length from one end, but when a weight of 10 lbs. is placed at the shorter end, the prop must be moved 2 feet nearer that end; find the weight of the beam. *Answer.* $23\frac{1}{3}$ lbs.

* 11. A force of 20 lbs. is applied at right angles to a lever 10 inches long 3 inches from one end, which is fixed; what force, acting at an angle of 45° at the other end, will balance it? *Answer.* 8·485 lbs.

* 12. In a lever of the first order, P = 3 cwt., R = $7\frac{1}{2}$ cwt., the angle between them 70°; find the strain on the fulcrum (cos. 70° = ·34202). *Answer.* 8·98 cwt.

* 13. In a lever of the second order, P = 120 lbs., R = 385 lbs., the angle between them 75°; find the strain on the fulcrum (cos. 75° = ·25882). *Answer.* 431·9 lbs.

* 14. In a lever of the third order, P = 200 lbs., R = 40 lbs., the angle between them 45°; find the strain on the fulcrum. *Answer.* 230 lbs.

15. A lever of the first order is 10 inches long; what are the arms, so that P may move 3 inches while W moves $1\frac{1}{2}$ inches? *Answer.* $6\frac{2}{3}$ and $3\frac{1}{3}$ inches.

16. A mass to be weighed, when put into one scale of an untrue balance appears to weigh $2\frac{1}{4}$ lbs.; when put into the other scale it appears to weigh 4 lbs.; determine its true weight. *Answer.* 3 lbs.

17. A substance is weighed from both arms of an untrue balance, and its apparent weights are 9 lbs. and 4 lbs.; find the ratio between the arms. *Answer.* 2 : 3.

18. Describe the common steelyard, and show that the graduations are equal.

19. Show how to graduate a steelyard of which the fulcrum is movable (Danish Balance).

20. A rod of uniform density, 6 feet long, has a weight of 2 lbs. tied to one end; it will balance on a fulcrum 6 inches from that end; what is the weight of the rod? *Answer.* $\frac{2}{5}$ lb.

THE WHEEL AND AXLE.

116. **The Wheel and Axle** is a modification of the lever, specially useful when the weight has to be moved through a great distance, as when water is to be raised from a well,

stones to be raised to the top of a wall, or an anchor to be drawn up to the deck of a ship. It consists of a cylinder or axle supported on a stand by a pivot at each end, capable of turning round its central line, and having attached to it a *wheel* concentric with it. Fig. 44 represents the wheel and axle in section. The resistance or weight, R, is attached to a rope wound round the axle, and the power P, to a cord passing round the wheel. The action at any moment is precisely that of a lever, of which the fulcrum is C, the common centre of the wheel and axle, and the arms A C, B C, the radii of the wheel and of the axle. The radii come into action successively, and the power and weight always act at right angles to their arms.

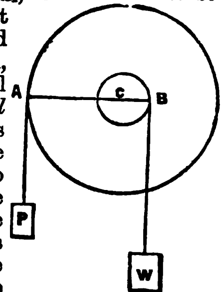


FIG. 44.

To ensure equilibrium, the moments about C must be equal: *i.e.*, $P \times AC = W \times BC$. Hence the condition of equilibrium for the wheel and axle is—

The power multiplied by the radius of the wheel is equal to the resistance multiplied by the radius of the axle, or **P : W :: radius of axle : radius of wheel.**

117. Instead of a complete wheel, only a few radii are sometimes employed. In the *Windlass*, P is sometimes applied to a single radius with a handle at its extremity, called a *winch*; but as power applied to such a handle is not constant, a second handle placed in an opposite direction at the other end of the axle is usually employed, and these handles are worked by two persons.

118. In the *Capstan* the axis is vertical, and is made to revolve by bars inserted into holes for the purpose. It is much used in ships for hauling in anchors, &c., and has the advantage that a large number of men can be combined in working it, whereas the windlass seldom admits of more than two; besides, it occupies little space, as the spikes used in working it are removed when not in use.

119. **Differential Axle** or **Chinese Windlass.**—The advantage of the wheel and axle may evidently be increased in two ways: (1) By increasing the size of the wheel; (2) By diminishing the size of the axle. But both these modes of improvement have limits. For on the one hand we

may make the wheel so large as to be unwieldy, or on the other hand we may so weaken the axle that it shall not be fit to support the weight. *The Differential Axle,* contrived to meet this defect, has an axle consisting of two portions of different thickness, and its mechanical effect depends not on the absolute thickness of the axle, but on the difference between the two parts. (Fig. 45.)

By each complete revolution of the winch handle the cord is rolled on the larger part, and unrolled from the smaller; it is thus shortened by the length of the larger circumference, and lengthened by the length of the smaller circumference; altogether, therefore, it is shortened by the difference of the two circumferences. The weight is consequently raised through a distance equal to half this difference. Thus, while P describes the circumference of the wheel, W moves through a space equal to half the difference of the circumferences of the axle. The power and weight are inversely proportional to the distances through which they move, and as the radii have the same proportion as the circumferences,

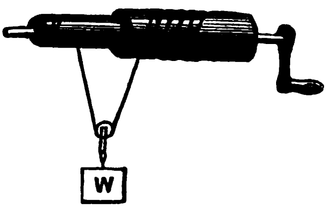


FIG. 45.

$$\frac{W}{P} = \frac{\text{radius of wheel}}{\frac{1}{2}\text{ diff. radii of axle.}} \therefore \mathbf{P \times its\ arm} = \frac{\mathbf{W}}{2}(\mathbf{r_1} - \mathbf{r_2}).$$

The efficiency of the machine can thus be made as great as we please without making it either too cumbrous or too weak, for the efficiency depends on the difference of the radii of the axle, not on their absolute magnitude.

120. **Toothed Wheels.**—A toothed wheel is a circular plate of metal (rarely of wood) having its edge cut into *teeth* all round. When two such wheels are placed so that a tooth of one fits into the space between two teeth of the other, then if one wheel be turned the other will turn also. In order to fit in thus, the teeth in the two wheels must be equal in size, and therefore the number of teeth will bear the same ratio as the circumferences or radii of the wheels. The wheels are really two constant levers, and P : W : : radius of W's wheel : radius of P's wheel; or, since the number of teeth is proportional to the radius, P × number of teeth in P's wheel = W × number of teeth in W's wheel.

The usual arrangement is that each wheel works with an axle. As will be seen from Fig. 46 this is really a combination of levers, and P × product of the alternate arms = W × product of remaining arms; or,

P × product of number of teeth in all the wheels = W × product of number of teeth in all the axles.

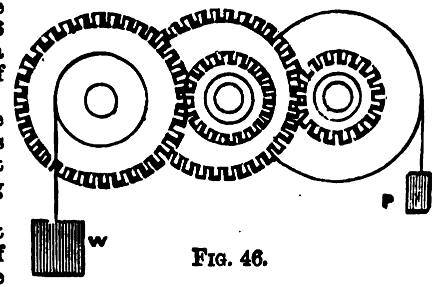


FIG. 46.

Sometimes teeth are dispensed with, and the wheels work by friction; this is possible, however, only where the resistance to be overcome is slight. And sometimes the wheels are connected by bands passing over them; this makes a very smooth, noiseless action.

121. **Hunting Cog.**—If the number of teeth in a wheel be a multiple of the number of *leaves* in the *pinion* (as the small wheel on the axle is called), each tooth of the wheel will always come into contact with the same tooth of the pinion, and if there is any defect in the construction of a particular tooth the wear and tear will be very unequal. To avoid this, it is usual to add to the wheel one tooth more than an exact multiple of the number in the pinion; this ensures that each tooth of the pinion will come into contact with every tooth of the wheel in succession. Thus, if there be 100 teeth in the wheel, and 11 in the pinion, each tooth of the pinion will come into contact with any one tooth of the wheel as often as with any other. The tooth so added is called the *hunting cog.*

122. Wheels are divided into crown, spur, and bevelled gear. The *crown wheel* has its teeth perpendicular to its plane; the *spur wheel* in its plane; the *bevelled wheel* at an oblique angle to its plane. *Bevelled wheels* are employed to change the direction of motion (Fig. 47). They are frusta of cones channelled into teeth to fit each other, and placed at whatever angle is required.

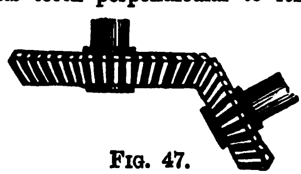


FIG. 47.

123. When water is the power employed, the wheel on which it acts has along its rim, boxes called *buckets*, instead of teeth. The water falling into these buckets moves the wheel round until the buckets are turned mouth downwards, when it escapes; and as new buckets are successively acted on, the wheel is kept in constant and uniform motion, and works by its axle on a toothed wheel connected with the machinery to be moved.

124. *Wheels of Carriages.*—In carriages, high wheels are preferable to low. They are more easily drawn over obstacles, whether heights or ruts. A small wheel will sink into a rut which would affect a large one very slightly; and when a large wheel encounters a height, such as a stone, it requires to be raised, with its load, along a much more gentle incline than a small wheel. Wheels are sometimes *dished*, *i.e.*, the spokes are made to incline outwards. When they get into a rut the lower spokes become more nearly vertical, and therefore are best able to resist the pressure, but in ordinary circumstances dished wheels are not advantageous.

125. *The Fusee of a Watch* depends on the principle of the differential axle. The moving power is a spring, which by its elasticity causes a cylinder to the inside of which it is attached to revolve; this acts by a chain attached to its outer surface on the fusee. The fusee is an axle of varying thickness, and

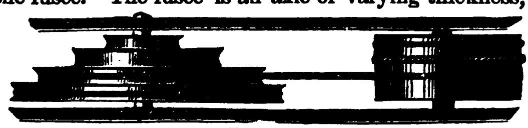


FIG. 48.

is so arranged that when the power of the spring is greatest, *i.e.*, soon after being wound up, its leverage is least; and as the action of the spring becomes more feeble, the leverage on the fusee increases. Thus the fusee is turned round uniformly, and imparts a uniform motion to the works of the watch.*

EXERCISES.

1. If the radius of the wheel be $3\frac{1}{2}$ feet, of the axle 5 inches, what force will be required to balance a resistance of 1 ton? 2240 lbs. × 5 ÷ 42 = $266\frac{2}{3}$ lbs. *Answer.*

2. A weight of 25lbs. balances 212lbs., and the radius of the wheel is 28 inches; find the radius of the axle. 3·3 inches. *Answer.*

* The groove of the fusee is really a spiral, and not as represented in the figure.

3. The radius of the axle in a windlass is 6 inches, length of the handle 2 feet 3 inches; what force will balance 850 lbs.? 27 : 6 :: 850 : P $= \frac{850 \times 2}{9} = 188\frac{8}{9}$ lbs. *Answer.*

4. In the differential axle the radii are 6 and 8 inches; what resistance will a force of 28 lbs., applied to a handle 25 inches long, overcome? 1 : 25 :: 28 : R $\therefore$ R = 700 lbs. *Answer.*

5. A weight of 17 lbs. just balances a weight of 79 lbs. on a wheel and axle; what will be the radius of the axle if that of the wheel be 17 inches? *Answer.* 3·658 inches.

6. The radius of a wheel is 25 inches, that of its axle $2\frac{1}{2}$ inches; what weight will a power of 17 lbs. applied to the wheel balance? *Answer.* 170 lbs.

7. The circumferences of the two parts of a differential axle are 30 and 27 inches, and the length of the handle to which the power is applied is 16 inches; find what force will raise a weight of 1 ton attached to the axle. *Answer.* 33·4 lbs.

8. In a combination of three wheels, with 80, 67, and 57 teeth, and three pinions with 12, 10, and 16 leaves, find the power required to support a ton weight. *Answer.* 14·077 lbs.

9. Find what weight 27 lbs. would balance in a combination of four wheels with 38, 74, 56, and 48 teeth, and four pinions with 12, 17, 21, and 10 teeth. *Answer.* 4763·86 lbs.

THE INCLINED PLANE.

126. **The Inclined Plane** is a plane surface inclined to the horizon at any angle. If a plane is horizontal, the whole weight of any body placed on it is borne by the plane; if it be vertical, a body placed in contact with it will not be supported by it at all. If the plane be inclined, part of the weight is borne by the plane, and part is uncounteracted and tends to make the body move down the plane; a body can therefore be supported on an inclined plane by a force less than its own weight, and the amount of this force will evidently depend on the slope or inclination of the plane.

127. Let A B C (Fig. 49) be a section of an inclined plane: A B is called the *length* of the plane, A C the *base*, and B C the *height*. The *inclination* of the plane is denoted by the angle B A C, or i, or by the ratio $\frac{BC}{AB} = \frac{\text{height.}}{\text{length.}}$ We shall consider the relation between the power and the weight in three cases.

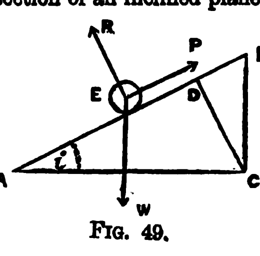


FIG. 49.

128. (1) When the power acts parallel to the length of the plane (Fig. 49), the weight is acted on by three forces to preserve equilibrium, viz., its own weight, W, acting downwards, the power, P, and the reaction of the plane R. Draw C D perpendicular to A B: then in the triangle B C D, C D is parallel to R, D B to P, and B C to W; therefore, by the Triangle of Forces $\frac{P}{W} = \frac{DB}{BC}$: and $\frac{R}{W} = \frac{CD}{BC}$. But the triangles A B C, B C D are equiangular to one another $\therefore$ by Euc. VI. 4,

$$\frac{DB}{BC} = \frac{BC}{AB}\text{; and } \frac{CD}{BC} = \frac{AC}{AB}. \quad \therefore \frac{P}{W} = \frac{BC}{AB} \text{ : and } \frac{R}{W} = \frac{AC}{AB}.$$

Hence the power multiplied by the length of the plane is equal to the weight multiplied by its height.

129. (2) When the power acts parallel to the base (Fig. 50) draw E F in same straight line with R. Then in the triangle E D F, the sides E D, D F, F E, are respectively parallel to W, P and R, therefore, as in the preceding article,

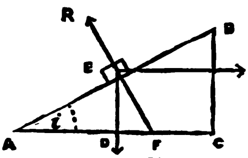


FIG. 50.

$$\frac{P}{W} = \frac{BC}{AC} \text{ and } \frac{R}{W} = \frac{AB}{AC}$$

Hence the power multiplied by the base of the plane is equal to the weight multiplied by its height.

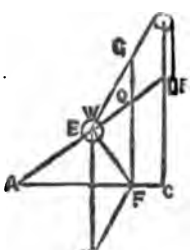


FIG. 51.

*130. (3) When the power acts in a direction making any angle ϕ with the length of the plane (Fig. 51), W is acted on by three forces in equilibrium, P, W, R. These forces are severally parallel to the sides E G, G F, F E, of the triangle E G F;

$$\therefore \frac{P}{W} = \frac{EG}{GF} = \frac{\sin. i}{\sin.(90+\phi)} = \frac{\sin. i}{\cos. \phi}$$

$$\therefore P \cos. \phi = W \sin. i.$$

Hence the power multiplied by the cosine of the angle it makes with the plane, is equal to the weight multiplied by the sine of the inclination.

131. In the lever we have an example of a fixed point furnishing the efficient support to the power, in the wheel and axle a fixed line, in the inclined plane a fixed surface.

132. When two bodies support each other on inclined planes which have a common height, they are to each other as the lengths of the planes on which they rest. For the force required to support W on A B $= W.\frac{BD}{AB}$; and that required to support P on BC is $=P.\frac{BD}{BC}$; and

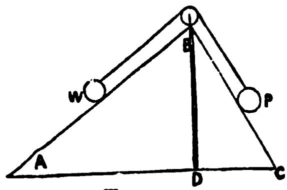


FIG. 52.

these forces are equal $\therefore P.\frac{BD}{BC} = W.\frac{BD}{AB}$;

$$\text{Or, } P \times AB = W \times BC; \text{ or, } P : W :: BC : AB.$$

EXERCISES.

1. A road has a rise or *gradient* of 1 in 13 : find its inclination.
   The sine of the angle of inclination is $\frac{1}{13}$ or ·07692 ; looking in the table we find that the angle corresponding to this sin is 4° 25′ ; this is therefore the inclination.
2. The inclination of a plane is 5° ; what is the gradient ?
   The sine of 5° is ·08715 ; the plane therefore rises ·08715 in 1, and the gradient is $\frac{1}{.08715}$ or 1 in 11·5.

3. If the force required to draw a wagon on a horizontal road be $\frac{1}{22}$ of its weight, what force is required to draw it up an incline of 1 in 30? *Answer.* ·0782 of its weight.

4. If the force required to draw a train on a level road be $\frac{1}{280}$ of its weight, what will it be when the road rises 1 in 45. *Answer.* ·025792 of its weight.

5. What force is required to roll a cask weighing 1,000 lbs. up a plank 12 feet long into a cart 3 feet high ? *Answer.* 250 lbs.

6. If the angle ϕ (Fig. 51) be 30°, i 20°, weight 3 cwt., what is the power (cos. 30° = ·86602 : sin. 20° = ·34202) ? *Answer.* 1·185 cwt.

7. A wagon weighing 3 tons rests on a plane rising 1 in 24 ; find the pressure on the rails. *Answer.* 2·9974 tons.

8. Show how high wheels are more easily drawn over obstacles than those that are low.

9. What power will sustain 1 ton on a plane whose gradient is 1 in 7, the power acting parallel to the base ? *Answer.* 2·8867 cwts.

10. A weight, W, rests on a smooth plane inclined 45° to the horizon ; required the least horizontal force that will suffice to sustain it. *Answer.* W.

11. What horizontal force is necessary to sustain a weight of 100 lbs. on a plane inclined at an angle of 30° to the horizon ? *Answer.* 57·736 lbs.

12. A weight of 300 lbs., resting on a plane of 60° inclination, balances (by means of a string passing over the common vertex of the two planes) another weight resting on a plane of 30° inclination; find the weight. *Answer.* 519·6 lbs.

13. A cask weighing 6 cwt. is to be raised into a waggon 3 feet high; find the force necessary to roll it up a plank 12 feet long. *Answer.* $1\frac{1}{2}$ cwt.

14. Describe accurately the forces by which the wheel of a carriage in motion is made to revolve on its axle.

15. If a body rests on a smooth plane inclined to the horizon at 30°, find the force necessary to hold the body, its direction making an angle of 45° with the inclined plane. *Answer.* ·7071 W.

THE WEDGE.

133. **The wedge** is a solid triangular prism of some hard material, used generally for separating the parts of another body. The thin end is introduced into a cleft in the body to be split, and the wedge is urged forward by pressure or blows applied to the back. Let A B C represent a section of a wedge; the power P is applied at right angles to the base A B, and counterbalances the resistances Q, R, which may be supposed to act at right angles to the sides of the wedge, A C, B C. The directions of these forces meet in a point O, and they may be regarded as three forces in equilibrium at O. Since the forces are perpendicular to the sides of the triangle A B C, they are proportional to the sides

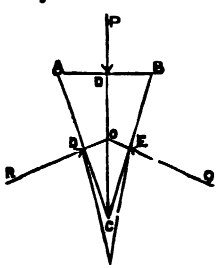


FIG. 53.

$$\therefore P : Q : R :: AB : BC : CA.$$

If Q and R are equal, as is usual, the wedge is called *isosceles*, 2 R is then the total resistance; and P : 2 R : : A B : 2 A C or as A D : A C. Let the angle A C D be called a.

$$\therefore \frac{P}{2R} = \frac{AD}{AC} = \sin a \therefore P = 2R \sin a.$$

From the proportion P : 2 R : : A B : 2 A C, we find that the power is to the total resistance as the back is to twice the length of the wedge.

This would hold good if the wedge were acted on by pressure alone; but it is usually acted on by blows which develop great force in very short time, and whose effect cannot be accurately calculated; besides, the friction is enormous.

All cutting instruments, as knives, hatchets, chisels, nails, and needles, are modifications of the wedge. It is frequently employed to produce great compression, also sometimes to tighten the parts of a structure.

THE SCREW.

134. **The Screw** (Fig. 54) consists of a solid cylinder A B, having on its surface a raised spiral line called the *thread*, and a hollow cylinder, C D, with a corresponding *groove* cut in its surface. The threads of the solid screw fit into the grooves of the hollow one. When either the solid or hollow screw is held firm and the other turned round, the latter, with anything attached to it, moves in the direction of the axis of the cylinder. The thread may be regarded as an inclined plane wound round the cylinder; the length of the plane is the length of the thread once round the cylinder, and its height is the distance between two threads, which is called the *pitch* of the screw. The power, P, is applied horizontally at the end of a long lever to move the weight, W, placed on an inclined plane, vertically. The plane has for its height h, the distance between two threads, called the pitch of the screw, and for its base $2\pi r$ = the circumference of the circle described by the power. Hence $P : W :: h : 2\pi r$.

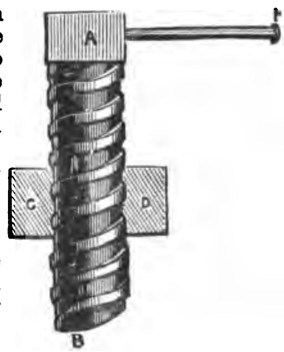


FIG. 54.

Practically the screw is a combination of the movable inclined plane and the lever. It is a very powerful machine, and has many useful applications. Screws are left-handed or right-handed, according to the direction of the spiral.

135. **Hunter's Differential Screw.**—The efficiency of the screw may be increased by increasing the length of the lever on which the power acts, or by diminishing the pitch. But this may make the lever inconveniently long, or the thread too weak, and therefore a form is adopted similar in principle to the differential axle. (Fig. 55.) Two screws are employed, differing slightly in pitch; that of smaller pitch works inside the other, and is attached at its other extremity to a bar or

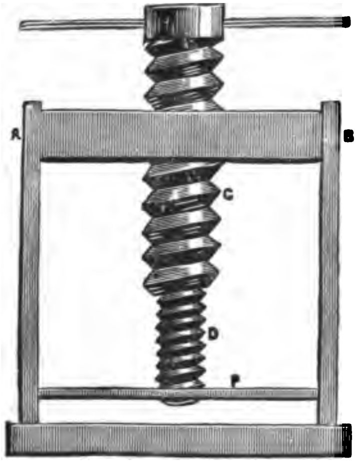


FIG. 55.

plate P, which can move up or down but cannot turn round. The hollow screw, A B, is fixed. When the handle is turned round once, the larger screw is lowered through a distance equal to its pitch; but the smaller screw is raised with reference to the larger through a distance equal to its pitch; hence the bar and the body connected with it are lowered through a distance equal to the difference of the two pitches. Therefore P $\times$ circumference described by its point of application $=$ W $\times$ difference of pitches.

$$P \times 2\pi r = W(h_1 - h_2).$$

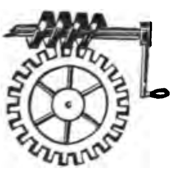


FIG. 56.

136. *Endless Screw.*—A few threads of a screw are sometimes employed to drive a toothed wheel; and as the action of such a screw may be continued indefinitely, it is termed the endless screw. (Fig. 56.) In this machine we have a combination of the wheel and axle and the screw; and P : W : : pitch of screw $\times$ radius of axle : circumference described by P $\times$ radius of wheel.

137. *Micrometer Screw.*—For small measurements a delicate screw, called the micrometer screw (Fig. 57) is sometimes used. If the pitch of the screw be $\frac{1}{10}$ inch, then the point of the pencil, *p*, attached to its extremity will be advanced $\frac{1}{10}$ inch by a complete turn of the handle; by half a turn $\frac{1}{20}$ inch; by $\frac{1}{10}$ of a turn $\frac{1}{100}$ inch; and in this way exceedingly small distances may be estimated.

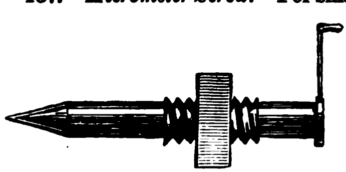


FIG. 57.

EXERCISES.

1. What is the ratio between P and R in a wedge whose back is 6 inches and length 12 inches? *Answer.* 1 : 4.

2. Find the ratio of P to R in a wedge whose angle is 20° (sin. 10° = ·17365). *Answer.* ·3473 : 1.

3. If the pitch of a screw be $\frac{1}{10}$ inch, and the power describe a circle of radius 6 inches, what is R if P be 10 lbs.? *Answer.* 3769·92 lbs.

4. The power describes a circle of 2 feet circumference; find the resistance which a power of 12 lbs. sustains, the pitch being $\frac{3}{8}$ inch. *Answer.* 768 lbs.

5. The pitch is $\frac{1}{4}$ inch; what power moving in a circle of 18 inches circumference will overcome 1 ton? *Answer.* $\frac{5}{18}$ cwt.

6. A power of 1 lb. moving in a circle whose circumference is 20 inches sustains a resistance of 36 lbs.; find the pitch of the screw. *Answer.* $\frac{5}{9}$ inch.

7. Describe the endless screw, and show what weight will be sustained by a power of 40 lbs. by means of an endless screw in which the winch is 20 inches long, the radius of the axle 2 inches, and the number of teeth in the wheel 80. *Answer.* 32,000 lbs.

8. If the intervals between the threads of a screw be $\frac{1}{29}$ inch, and the circumference of the circle described by the power 100 feet; what weight will a power of 20 lbs. sustain? *Answer.* 696,000 lbs.

9. In a differential screw the pitches are $\frac{1}{4}$ and $\frac{1}{3}$ of an inch, and a power of 1 lb. acts at the end of a lever 12 inches long; find the resistance which it will balance. *Answer.* 904·78 lbs.

10. In a micrometer screw the handle is 6 inches long, and the pitch of the screw is $\frac{1}{4}$ inch; the handle is moved through an angle of 60°; through what distance does the pencil advance? *Answer* $\frac{1}{24}$ inch.

THE PULLEY.

138. **A Pulley** is a wheel or disc turning freely on an axle, and having a groove on its circumference, along which passes a *cord*, which is really the efficient agent, no mechanical advantage being derived from the pulley itself. The disc is called a *sheaf* and turns in a case call a *block*. The pulley itself is really a constant lever with equal arms, and the power and weight always act at right angles to the arms in action.

The cord is supposed to be perfectly flexible and inextensible. These conditions, of course, can never be really obtained, and we must in practice make allowance accordingly. The cord undergoes the same tension in every part, and this tension is equal to the power applied at its extremity.

Pulleys are either *fixed* or *movable*.

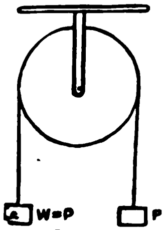


FIG. 58.

A fixed pulley is useful only in changing the direction of motion. This is often very important. For instance, in raising a heavy weight, it is much more convenient to pull downwards, *i.e.*, in a direction in which the weight of the agent is effective, than in the contrary direction. Here, **W** = **P.**

139. **Single Movable Pulley.**—In A and B the same tension exists at every part of the cord; this tension is equal to P, the force applied at one end. P is supported by one part of the string, W by two; but each bears a tension equal to P; therefore, **W** = 2 **P.** The pulley here is really a lever of second order: P applied at one side, W in middle, F at other side. P has arm = 2 W's arm ∴ W = 2 P.

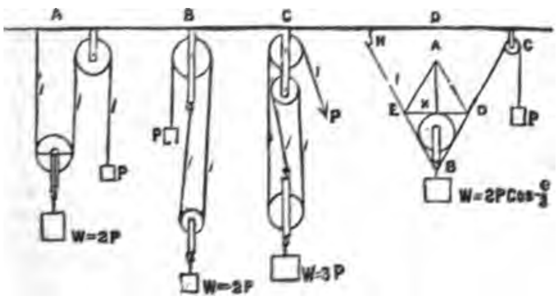


FIG. 59.

In C, the power is supported by one part of the thread, the weight by three; therefore, **W** = 3 **P.** This arrangement gives the greatest advantage possible with one pulley.

In A, B, and C, the parts of the cord which support the weight are parallel; when this is not the case, the effect is diminished. In D the two parts of the cord are inclined at an angle H B C; call this angle θ. The tensions in H B and C B are each equal to the power. Draw E D parallel to H C, and complete the parallelogram A E B D; A B represents W on the same scale on which B D or B E represents P.

Therefore since A B D is evidently $= \frac{\theta}{2}$, we have

$$D B : B x :: P : \frac{W}{2} \therefore \frac{W}{2} = \frac{P.B x}{D B} = P \cos. ABD \therefore \mathbf{W} = 2 \mathbf{P} \cos. \frac{\theta}{2}$$

140. **First System of Pulleys.**—The arrangement at H, Fig. 60, in which each pulley hangs by a separate cord, and all the cords are parallel, is called the ***First System of Pulleys.*** Here P is borne by one part of the cord, the first movable pulley by two, the tension in each being equal to P; the second movable pulley by two, the tension in each of which is = 2 P; the third pulley by two, the tension of each being 4 P; and the weight by one, the tension of which is 8 P. Therefore, in the first system, each cord added doubles the effect, and $\mathbf{W = P \times 2^n}$.

In the arrangement shown at I (Fig. 60) each cord passes over a fixed pulley and under a movable one, and is, therefore, divided into three portions. The tension in every part of the first cord is evidently = P; in the second cord 3 P; in the third 9 P; in the cord which bears the weight 27 P. Each cord added trebles the effect; and

$$\mathbf{W = P \times 3^n}$$

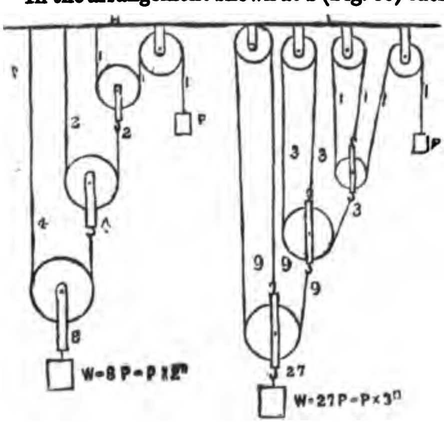


FIG. 60.

141. **Second System of Pulleys.**—In this system, represented at E and F (Fig. 61) the same cord passes round all the pulleys, and the parts of it between the pulleys are parallel. Unless otherwise stated, it is assumed in the calculations that the portions of the string not in contact with the pulleys are parallel, but in practice this may not invariably be the case. In E, the weight is 4 times, in F 6 times the power. If we had four movable pulleys, the weight would be 8 times the power, and generally, if n be the number of movable pulleys, $\mathbf{W = P \times 2}n$.

If the string were fastened to the lower block the weight would be supported by $2n + 1$ parts of it, and we should have

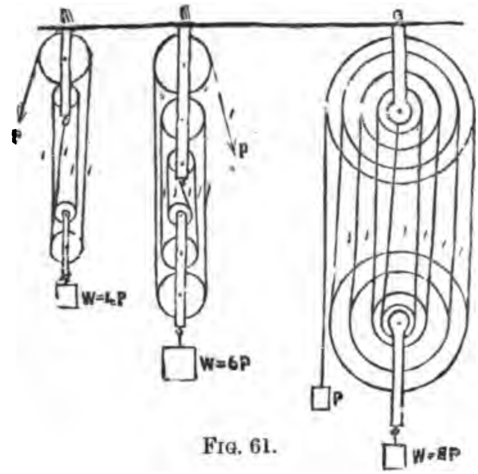


FIG. 61.

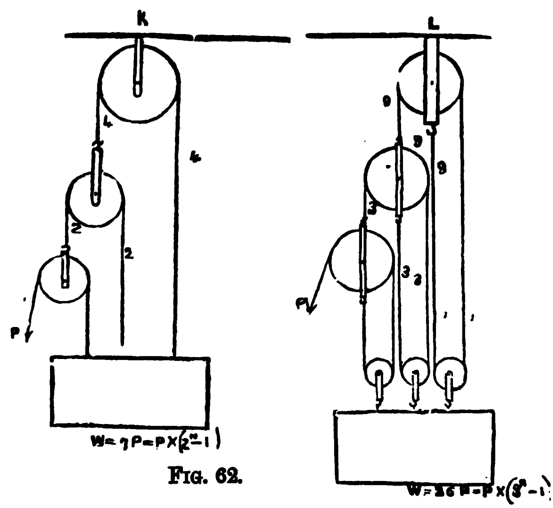


FIG. 62.

$W = (2n + 1) P$. If we call the number of portions of string passing from the lower block N, we shall have $\mathbf{W = N P}$. In all cases, the weight of the lower block is included in W.

In G, *White's Pulley* is shown, in which the fixed pulleys are all in one block, and the movable pulleys in one block. The diameters vary in such a way that they all turn at once, and thus avoid the friction inseparable from the other cases. This pulley, however, is not much employed; it is hard to prevent the cord from slipping from the grooves and getting tangled. The formula is still the same.

142. **Third System of Pulleys**, K, Fig. 62.—In this system, each cord is attached to the weight, and passes over one movable pulley; and all the cords are parallel. (In this case the fixed pulley helps to support the weight in the same manner as the movable pulleys do, and must, therefore, be included in counting the number of pulleys. The weight is less by once the power than in the first system. Each cord added *almost doubles the effect;* and

$$\mathbf{W = P \times (2^n - 1)}.$$

In the modification of the Third System shown at L, each cord passes round two pulleys, and is divided into three parts. The weight is less by once the power than in the modification of the First System, I, Fig. 60. Each cord added *almost trebles the effect;* and $\mathbf{W = P \times (3^n - 1)}$.

The Spanish Burton, M, N, is sometimes employed in raising heavy wine casks.

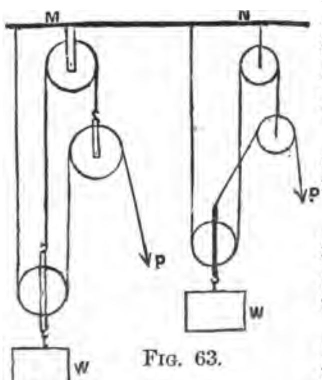


FIG. 63.

143. The weights of the pulleys themselves modify the effect. We shall now examine how they are to be taken into account.

In the First System of Pulleys, Fig. 60, each part of the cord which supports the weight has a tension equal to half the weight, W, + half the weight of the lowest pulley, w_1; each part of the next cord has a tension equal to half this tension and half the weight of the second pulley, w_2; each

part of the last cord bears half this weight and half the weight of the third pulley, w_3.

$$\therefore \text{tension on first cord} = \frac{W + w_1}{2}.$$

$$\text{,, second ,,} = \frac{W + w_1}{4} + \frac{w_2}{2}.$$

$$\text{,, third ,,} = \frac{W + w_1}{8} + \frac{w_2}{4} + \frac{w_3}{2} = P.$$

To find the Power, therefore, add the Weight to the weight of the lowest pulley, and divide by 2; to the quotient add the weight of the next pulley and divide by 2; to this quotient add the weight of the third pulley and divide by 2; and continue the process as many times as there are movable pulleys. To find the Weight, double the Power and subtract the weight of the uppermost pulley; double the result and subtract the weight of the next pulley; double this result and subtract the weight of the next pulley; and continue the process as often as there are movable pulleys.

In the system I (Fig. 60) the same processes are employed, except that 3 is used as divisor and multiplier instead of 2.

In the Third System (Fig. 62) the tension of the first cord $=P$; of the next, $2\ P+w_1$; of the next, $4\ P+2\ w_1+w_2$; of the next, $8\ P+4\ w_1+2\ w_2+w_3$, &c. Now, as the weight is supported by all these cords conjointly, it must be equal to the sum of these tensions:

$$\therefore W=P+2\,P+w_1+4\,P+2\,w_1+w_2+8\,P+4\,w_1+2\,w_2+w_3, \text{\&c.}$$

Thus, if there are two pulleys, $W=3\ P+w_1$; if three, $W=7\ P+3\ w_1+w_2$; if four, $W=15\ P+7\ w_1+3\ w_2+w_3$, &c.

The same method holds good in the System at L (Fig. 62), except that 3 is used as multiplier instead of 2.

The various kinds of pulleys are much used on shipboard, for hoisting sails, raising cargo, and lowering into hold, etc.

EXERCISES.

1. In the First System of pulleys, what weight will a power of 12lbs. support by means of 5 pulleys? *Answer.* 384lbs.

2. In the First System, what power will support a weight of 15 cwt. by means of 3 pulleys? *Answer.* $1\frac{7}{8}$ cwt.

3. In the Second System, what weight will be supported by 3 cwt. with 4 pulleys in lower block, the string being fastened above? *Answer.* 24 cwt.

4. In the Second System, what power will be required to balance a weight of 1 ton with 4 pulleys? *Answer.* 2·5 cwt.

5. In the Third System, what weight will a power of 120lbs. support with 3 pulleys? *Answer.* 840lbs.

6. In the Third System, what power will balance a weight of 3 tons with 5 pulleys? *Answer.* $1\frac{29}{31}$ cwt.

7. In the System L (Fig. 62), what weight will be supported by 25lbs. with 7 pulleys? *Answer.* 54,650lbs.

8. In the System I (Fig. 60), what power will support 1 ton with 3 pulleys? *Answer.* $\frac{20}{27}$ cwt.

9. In Fig. 59. D, if the angle E B D be 60°, and P, 120lbs; find W. *Answer.* $120\sqrt{3}$.

10. Find the weight supported by 1lb. in each of the three systems with five pulleys. *Answer.* 32lbs.; 10lbs.; 31lbs.

11. In the First System what weight will be supported by a power of 10lbs. if there are four pulleys weighing 2, 3, 4, 5lbs. (commencing with pulley next the weight). *Answer.* 96lbs.

CHAPTER VI.

FRICTION.

144. If surfaces were perfectly smooth, the reaction of each would be only in the direction of a normal or perpendicular to the surface, and the slightest force would move one over another. But no surfaces are perfectly smooth; those which seem so will be found on examination by the microscope to be covered with small inequalities, hence they offer resistance in other directions as well as that of the perpendicular. The resistance which a surface thus offers to the passage of another over it is called **friction.** Friction usually prevents or destroys motion; but it may also be made the means of imparting motion. Thus, one wheel may be made to revolve by another rolling against it.

145. The Limiting Angle of Resistance, or Angle of Repose.—Let the bar A B (Fig. 64) be pressed in the direction of its length against the plane with a force represented by F A. Resolve F A into two forces F O, O A. F O is destroyed by the reaction of the plane A C; O A by friction, represented by A O. Let the angle B A C be gradually diminished. Whenever the component O A exceeds the friction of the plane, the beam will begin to slide along the plane. The angle B A D, between the beam and the normal to the surface, at which the sliding just begins, is the *angle of friction*, called also the limiting angle of resistance or angle of repose.

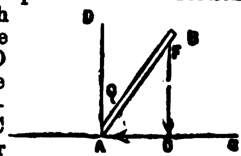


FIG. 64.

146. The angle of repose may be determined otherwise. Let a body (Fig. 65) rest on the plane A B in a horizontal position; raise the plane at the end A gradually until the body begins to slide; ϕ the angle of inclination of the plane A B is the angle of repose.

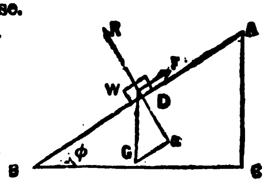


FIG. 65.

147. If force is exerted to move a body, friction opposes that force; if the force be exerted to stop a body already in motion, friction aids it. Thus, "friction always aids the weaker force."

148. When one surface slides over another the resistance is called *sliding* friction; when one rolls on the other, the resistance is *rolling* friction. Rolling friction is much less, with the same surfaces and pressure, than sliding; hence in vehicles wheels are employed which replace sliding friction by rolling, and thus lessen the motive force required. With the same object the axle of a wheel is sometimes supported on the circumferences of four small wheels called *friction-wheels*, two at each end. These move freely with the axle, and lessen the friction very much. In descending a steep hill the wheels of a carriage are retarded by a break or a chain, and thus the rolling is converted into a sliding motion,. and greater resistance is offered to the downward pressure. Breaks are similarly employed on railway trains to facilitate the stopping of the carriages.

149. **The Coefficient, Index, or Modulus of Friction** is the ratio which the friction in any case bears to the effective pressure. It is measured by the tangent of the angle of repose (Fig. 65), for the body is supported on the plane by the three forces—F friction, W its weight, R the reaction of the plane; these are proportional to the sides of the triangle D G a; and $\frac{F}{R}=\frac{G\,a}{a\,D.}=\frac{A\,C}{B\,C.}$ Therefore $\frac{\text{friction}}{\text{pressure}}=\frac{A\,C}{B\,C}=\tan.\ \phi$. The coefficient of friction is represented by the letter μ, hence $\mu=\tan.\ \phi$.

150. The coefficient of friction varies in value with the nature of the surfaces in contact. Thus, when raw wood is rubbed on raw wood, with the fibres in the same direction, $\mu=\frac{1}{2}$; if the fibres cross, about $\frac{1}{4}$; when metals are rubbed together, μ is about $\frac{1}{4}$; when wood slides on metals, $\frac{1}{5}$. After long rubbing the friction becomes less, and it may be still further diminished by greasing the surfaces.

151. **Laws of Friction.**—The laws of friction were very carefully investigated by M. Coulomb, a French officer, who obtained the following results:—

(1) The friction between two surfaces of the same kind is directly proportional to the pressure between them.

(2) The friction is independent of the extent of the surfaces in contact.

(3) The friction is independent of the velocity when a body is in motion.

(4) Friction varies with the nature of the surfaces in contact, and with their relative disposition, whether, *e.g.*, the fibres of wood cross or run in the same direction.

EXERCISES.

1. State the laws of friction. When a body can just rest on an inclined plane, find an expression for the coefficient of friction.

2. A body will just rest on a plane inclined at an angle of 30°; find the coefficient of friction. *Answer.* $\frac{1}{\sqrt{3}}$

3. A body placed on a rough horizontal plane requires a horizontal force equal to its own weight to overcome the friction. Supposing the plane gradually inclined, find at what angle the body will begin to slide. *Answer.* 45°.

*4. Find the least force that will drag a weight of 80lbs. along a rough horizontal plane, the friction being such as would just prevent the body from sliding down a plane of inclination 30°. *Answer.* 40lbs. inclined at 30°.

5. A body weighing 54lbs. is just set in motion on a rough horizontal plane by a horizontal force of 9lbs. If the force be withdrawn and the plane tilted up, at what inclination of the plane to the horizon will the body begin to slide? *Answer.* At the angle whose tangent is $\frac{1}{6}$.

CHAPTER VII.—KINEMATICS.

MOTION.

152. **Motion,** as we have seen (Art. 4), is change of position in space. The motion of any body is measured by means of its velocity.

Definition.—*The velocity of any moving body is the rapidity with which it moves. If the body passes through equal distances in equal times its velocity is said to be* uniform ; *but if not,* variable. *Uniform velocity is measured by the number of units of length passed over in a unit of time. Variable velocity is measured at any selected instant of time by the number of units of length which would be passed over in a unit of time, if during the whole of that unit the body were moving uniformly as it is moving at the instant under consideration.*

153. This definition requires some explanation. As motion is change of position in space, in order to know the motion of a body we must know either the actual space which it has passed through, or the space which it would pass through under certain conditions. But whilst it is *necessary* to know the space described, that knowledge is not *sufficient* to determine the motion ; as may be seen by an example. If we are told that a man has gone to a place a mile distant, we know the extent of his motion, but not its quality : for he may have walked the distance in 15 minutes or run it in 5 minutes. So that we must know the time as well as the space, if we are to know the motion fully. And there is yet a further condition : for if this man occupied 15 minutes in traversing the mile, he may have walked half way, then stopped 5 minutes, and run the remainder. We require, therefore, to know whether his speed was uniform ; and, if not, how it varied, before our knowledge of his motion is complete. If it was uniform, we may, according to our choice of units, describe his velocity as $5\frac{13}{15}$ feet per second, or 4 miles per hour, *although he did not go so far as 4 miles.*

154. When a velocity varies, no single number can represent it throughout the motion ; it has a separate value at each instant, which value is measured by the same number as would be required to express it, if it continued without further change.

155. It is often useful to find the **average or mean velocity** of a body during any time. This is obtained by dividing the

total space passed over by the number of units of time. Thus, if a body move in four successive seconds 12, 17, 13, 14 feet, the average velocity is $(12+17+13+14) \div 4 = 14$ feet per second.

156. In the case of a body moving uniformly, if v be the velocity per second, t the number of seconds, and s the space, then the space passed over in

$$\begin{array}{lll} 1 \text{ second} & = v \\ 2 \quad ,, & = 2v \\ 3 \quad ,, & = 3v \\ t \quad ,, & = tv \end{array}$$

The formula for uniform motion, therefore, is

$$s = vt$$

If the motion be uniform in a circle it may be measured by the angle subtended at the centre by the arc described by the body in one unit of time. Thus, if the arc described in 1 second, be called a, and the angle at the centre subtended by this arc ω, then the arc traversed in t seconds will be $a\,t$, and the angle subtended by it $t\,\omega$. The angle ω is usually expressed in *circular measure*, in which the unit of measurement is the angle subtended at the centre by an arc equal to the radius of the circle $= \frac{180^\circ}{\pi}$.

157. *Acceleration is the rate at which the velocity of a body increases. It is said to be* uniform *when the velocity is increased by equal increments in equal times; otherwise it is* variable. *Uniform acceleration is measured by the additional velocity acquired in each unit of time.*

Decrease of velocity is sometimes called retardation; but it is better considered as a negative acceleration. Thus if a body has a velocity of 5 feet at the end of the first second, 10 feet at the end of the next, and 15 feet at the end of the third second, the acceleration is uniformly 5 feet per second. If, again, at the ends of three successive seconds a body has velocities of 40, 37, 34 feet, the acceleration is negative, and amounts to 3 feet per second.

If we call the acceleration f, the number of seconds t, and the velocity at any time v,

$$\begin{array}{llll} \text{The velocity gained in} & 1 \text{ second} & = f. \\ ,, \qquad ,, & 2 \quad ,, & = 2f. \\ ,, \qquad ,, & 3 \quad ,, & = 3f. \\ ,, \qquad ,, & t \quad ,, & = tf. \end{array}$$

Hence $\mathbf{v = ft.}$ (1)

158. *To find the space described.*—We shall employ the principle of mean velocity referred to above.

Suppose a body at rest to be put in motion and to have imparted to it a uniform acceleration f. Since the velocity in the first second begins at 0 and increases uniformly up to f, the average velocity during that second is $\frac{1}{2}f$, and therefore the space described in that second $= \frac{1}{2}f$. In the next second the velocity increases from f to $2f$, therefore the average velocity, and consequently the space passed over in that second, is $\frac{f+2f}{2} = \frac{3f}{2}$. Hence

Space in 1st second	$= \frac{f}{2}$	in 1 second	$= \frac{f}{2}$
„ 2nd „	$= \frac{3f}{2}$	2 „	$= \frac{4f}{2}$
„ 3rd „	$= \frac{5f}{2}$	3 „	$= \frac{9f}{2}$
„ „	$= \frac{(2t-1)f}{2}$	t „	$= \frac{ft^2}{2}$

$$\text{Hence} \quad \mathbf{s = \tfrac{1}{2} f t^2} \quad (2)$$

We have seen that $v = ft$, and $s = \frac{1}{2}ft^2$. Square the former, and multiply both sides of the latter by $2f$; we thus get

$$v^2 = f^2 t^2$$

$$2fs = f^2 t^2, \text{ and therefore}$$

$$\mathbf{v^2 = 2fs}. \quad (3)$$

159. Another method. *If* **s** *be the space described from rest in time* **t** *by a particle under the action of a uniform acceleration* f, $s = \frac{1}{2}ft^2$.

Let the time t be divided into n intervals, each equal to , so that $n\tau = t$; then the velocities at the *beginning* of these successive intervals will be—

$$\begin{array}{ccccc} 1 & 2 & 3 & \ldots & n \\ 0 & f\tau & 2f\tau & \ldots & (n-1)f\tau; \end{array}$$

and at the *end* of the same intervals, the velocities will be—

$$f\tau \quad 2f\tau \quad 3f\tau \quad nf\tau.$$

Now, if the particle moved during each interval with the

velocity which it has at the beginning of that interval, the space described would be—

$$0\tau + f\tau\cdot\tau + 2f\tau\cdot\tau + 3f\tau\cdot\tau + \ldots (n-1)f\tau\tau$$

which is $= f\tau^2 + 2f\tau^2 + 3f\tau^2 + \ldots (m-1)f\tau^2$

$$= f\tau^2 \left\{1 + 2 + 3 + \ldots (n-1)\right\}$$

$$= f\tau^2 \frac{n(n-1)}{2}.$$

$$= \frac{ft^2}{n^2}\cdot \quad \frac{n(n-1)}{2}\left(\text{since } \tau = \frac{t}{n}\right) = \tfrac{1}{2}ft^2\left(1 - \frac{1}{n}\right)$$

Again, if the particle were to move during each interval with the velocity which it has at the end of the interval, the space described would be—

$$f\tau\cdot\tau + 2f\tau\cdot\tau + 3f\tau\tau + \ldots + nf\tau\tau$$

$$= f\tau^2\left(1 + 2 + 3 + \ldots + n\right) = \frac{n(n+1)}{2}f\tau^2$$

$$= \tfrac{1}{2}ft^2.\left(1 + \frac{1}{n}\right)$$

Since the velocity is continually increasing during the time t, the space *actually* described by the particle will be intermediate between these two values; *i.e.*, s lies between $\frac{1}{2}ft^2$ $\left(1 - \frac{1}{n}\right)$ and $\frac{1}{2}ft^2$ $\left(1 + \frac{1}{n}\right)$ however large n be taken.

But when n is taken indefinitely large $\frac{1}{n}$ becomes $= 0$; and hence $s = \frac{1}{2}ft^2$. The equations connecting s, f, v, t, are therefore—

$$v = ft \quad \ldots (1)$$

$$s = \tfrac{1}{2}ft^2 \quad \ldots (2)$$

$$v^2 = 2fs \quad \ldots (3)$$

160. If the body, instead of starting from rest, had an initial velocity $= u$, and then acquired the acceleration f,

Velocity at beginning of		1st	second	$= u$.
"	end	"	"	$= u + f$.
"	"	2nd	"	$= u + 2f$.
"	"	last	"	$V = u + tf$.

Space described in 1st second $=u+\frac{f}{2}$ in 1 second $=u+\frac{f}{2}$

" 2nd " $=u+\frac{3f}{2}$ " 2 " $=2u+\frac{4f}{2}$

" 3rd " $=u+\frac{5f}{2}$ " 3 " $=3u+\frac{9f}{2}$

" last " $=u+\frac{f(2t-1)}{2}$ " t " $=tu+\frac{ft^2}{2}$

Hence, with initial velocity, the equations become

$$\mathbf{v} = \mathbf{u} + \mathbf{f}\,\mathbf{t} \qquad (4)$$

$$\mathbf{s} = \mathbf{t}\,\mathbf{u} + \frac{\mathbf{f}\,\mathbf{t}^2}{2} \qquad (5)$$

$$\mathbf{v}^2 = \mathbf{u}^2 + 2\,\mathbf{f}\,\mathbf{s} \qquad (6)$$

In practice it will be best always to use these equations, putting $u = 0$, when the particle starts from rest.

161. **Composition of Velocities.**—If a body has several velocities imparted to it at the same time, it will actually move with some one velocity, which is the *resultant* of the velocities imparted to it. Thus, if a body have imparted to it at the same moment two velocities of 12 and 10 feet per second respectively in the same direction, its actual velocity will be their sum, 22 feet, in the same direction. If, on the other hand, the velocities impressed on the body be in opposite directions, the resultant will be a velocity of 2 feet in the direction of the greater. If the velocities be neither directly concurrent nor directly opposed, the resultant will be less than 22, and greater than 2. If the velocities imparted be not in the same straight line, the resultant velocity can be found by a very easy construction. If a body at A (Fig. 66) have imparted to it a velocity v, which would carry it to B in 1″, and also a velocity v', which would carry it to C in 1″; then at the end of the second it will be found at D, having moved along A D the diagonal of the parallelogram described on A B, A C, as adjacent sides. Hence the—

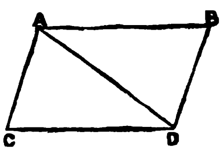


FIG. 66.

Parallelogram of Velocities.—*If two adjacent sides of a parallelogram represent in magnitude and direction the velocities*

impressed on a body, the resultant velocity shall be represented in magnitude and direction by the diagonal drawn through that point.

If more than two velocities are impressed on a body the resultant velocity can be found in the same way. Thus, if velocities of 6, 8, 10 are imparted in different directions, we may find the resultant of 6 and 8, and then compound that resultant with 10.

162. **Resolution of Velocities.**—Any velocity represented by a straight line can be decomposed into two velocities represented by two adjacent sides of any parallelogram of which that line is the diagonal. Thus A B can be decomposed into A D and A C (Fig. 67).

The following examples will make this clear:—

(1) A body has a velocity of 30 feet per second; resolve this into components at angles of 30° and 60°.

Let A B represent the velocity; make angles at A and B, of 60° and 30°; A C and A D will represent the components. A C is half of A B, therefore = 15, and A D = $\sqrt{30^2 - 15^2} = \sqrt{675}$ = 25·98 (Fig. 67).

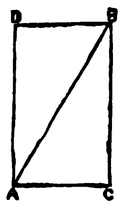


FIG. 67.

(2) Resolve a velocity of 50 into two equal components at right angles.

Here each component will evidently make with A B an angle of 45°. Therefore A C = $\sqrt{\frac{A\,B^2}{2}} = \sqrt{1250} = 35{\cdot}35$ feet.

The statements in this paragraph and the last apply to accelerations as well as velocities.

163. **Graphic Representation of Motion.**—We may represent by lines and areas the various elements of motion, time, velocity, and space. Thus, if we take A B, B C, &c. (Fig. 68), to represent units of time, and if the velocity at A be represented by the line Aa, at B by Bb, at C by Cc, &c., and if these lines be equal, we have a graphic representation of uniform motion. The space passed over in the first second may be represented by the rectangle Ab, in two seconds by Ac = 2 Ab, in five seconds by A G = 5 Ab.

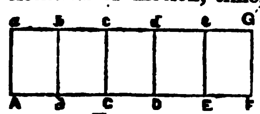


FIG. 68.

Again, let A (Fig. 69) denote the position of a body starting from rest; let A B, B C, &c., represent units of time, and B*b*, C*c*, &c., the velocity of the body at the end of the successive seconds, so that C*c* = 2 B*b*, D*d* = 3 B*b*, &c. The figure will then represent motion with uniform acceleration. The line A G is a straight line. The triangle A B*b* represents the space described in 1″, A C*c* in 2″, A D*d* in 3″, and so on.

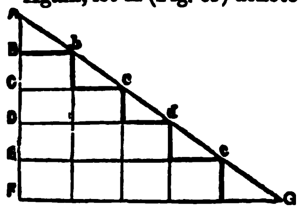


FIG. 69.

That is, space in 1″ $= \frac{f}{2}$; in 2″ $= \frac{4f}{2}$, &c.

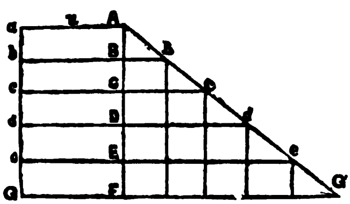


FIG. 70.

In Fig. 70, let A *a* represent the initial velocity *u*; the calculation will be the same as before, with the addition of *u* to the velocity, and *u* to the space for each second.

EXERCISES.

1. If a stream carries a boat forward 5 miles an hour, and the wind 7 miles, what is the actual velocity of the boat? *Answer.* 12 miles.

2. If the boat is urged by the wind 7 miles an hour up the stream, and carried back 5 miles an hour by the stream, find its actual velocity. *Answer.* 2 miles up stream.

3. If a ball be acted on at right angles by two forces imparting velocities of 3 feet and 4 feet per second, what is its actual velocity? *Answer.* 5 feet.

4. Reduce 300 yards per hour to unit velocities. *Answer.* $\frac{1}{4}$.

5. Reduce 45 miles per hour to unit velocities. *Answer.* 66.

6. Find the velocity per second of any point at the equator. *Answer.* 1527·7 feet per second.

7. One body moves south at the uniform rate of 9·8 metres per second, another east from the same point at the rate of 17·6 metres per second; both started at the same time; how far will they be asunder in 3 minutes? *Answer.* 3626 metres.

8. A body moves round a circle 30 inches in diameter in 4 seconds; find its velocity in feet, and also its angular velocity. *Answer.* 1·96 feet; 90° (or, expressed in *circular measure*, $\frac{\pi}{2}$ or 1·5708).

9. If a ball be dropped out of a railway carriage, in what direction will it seem to fall—(1) to a person in the carriage, (2) to a person standing on the railway bank?

10. A body is moving north-east with a velocity of 10 metres per second; how far does it go northward in 5 minutes? *Answer.* 2121·3 metres.

11. A body has an acceleration of 30 feet along a certain line; find its acceleration along a line inclined at an angle of 60° to the former. *Answer.* 15 feet.

12. A balloon is moving with a current of air uniformly 60 miles an hour east to west; a feather is dropped from it; what will be the motion of the feather as seen by a man in the balloon?

13. The velocity of a train increases uniformly. At one o'clock its velocity was 12 miles an hour, at ten minutes past one o'clock its velocity was 36 miles an hour; what was its velocity at $7\frac{1}{2}$ minutes past one o'clock? *Answer.* 30 miles.

*14. In accelerated motion, show that the space is proportional to the square of the time, when a body moves from rest under the action of a constant force.

15. A velocity of 20 feet per second is produced by a force acting for 2 seconds; calculate the acceleration. *Answer.* 10 feet per second.

16. A force acting for 10 seconds produces a velocity of 25 miles an hour; calculate the acceleration. *Answer.* $3\frac{2}{3}$ feet per second.

17. A force producing an acceleration of 12 feet per second acts for 17 seconds; find the velocity imparted. *Answer.* 204 feet.

18. In what time will a force producing an acceleration of 32 feet per second produce a velocity of 672 feet per second? *Answer.* 21 seconds.

19. A body under the influence of a constant force moves from rest through 500 feet in 20 seconds; find the acceleration. *Answer.* $2\frac{1}{2}$ feet per second.

20. If the space described be 3050 feet in 12 seconds; find the acceleration. *Answer.* 42·36 feet per second.

21. In what time will a body acted on by a force producing an acceleration of 12 feet per second move through 1 mile? *Answer.* 29·6 seconds.

22. Through what space will a body under an acceleration of 25 feet per second move in half a minute? *Answer.* 11250 feet.

23. What is the acceleration due to a force which imparts a velocity of 200 feet per second, after moving a body through 200 feet? *Answer.* 100 feet per second.

CHAPTER VIII.

MOTION OF FALLING BODIES.

164 Gravity may, generally speaking, be considered as a constant force. It therefore produces in falling bodies a uniformly accelerated velocity; and hence the three formulæ already established with reference to uniformly accelerated velocity will apply to the motion of falling bodies. The acceleration of gravity being called g, we have, when the body acted on starts from rest,

$$(1)\ v = gt.$$
$$(2)\ s = \tfrac{1}{2}gt^2.$$
$$(3)\ v^2 = 2gs.$$

If there is an initial velocity, u, these formulæ take the following form:—

$$(4)\ v = u \pm gt.$$
$$(5)\ s = ut \pm \tfrac{1}{2}gt^2.$$
$$(6)\ v^2 = u^2 \pm 2gs.$$

(Instead of these formulæ, we may use equations (4), (5), and (6) of Art. 160.) If the initial velocity is downward, we employ the positive sign; if upward, the negative. We proceed to explain how these laws have been determined experimentally.

165. It was formerly supposed that heavy bodies fall more quickly than light bodies, and ordinary experiences would seem to confirm this. For instance, if a leaden bullet and a feather be dropped from the same height at the same instant, the bullet will reach the ground first. But Galileo proved by very conclusive experiments made from the celebrated leaning tower of Pisa, that the difference is due to the resistance of the air, which is much greater relatively in the case of the feather than in the case of the bullet, and showed that when fairly tried all bodies fall with the same velocity. From this he inferred that gravity acts with the same force on all substances, and that in a vacuum all bodies of whatever shape would fall with the same velocity. This may be shown experimentally. Take a long glass tube and put into it small pieces of paper, sand, gold, &c. Invert the tube; the heaviest will reach the bottom first. Now let the air be pumped out of the tube, and the experiment tried again. All the bodies, light

and heavy, will fall in the same time. Again, the same fact may be shown without using an air-pump. Place a piece of paper, smaller than the coin, on a penny, and let the penny drop in a horizontal position: the paper will fall as fast as the penny; now the only thing removed from the paper being the resistance of the air, this shows that gravity imparts the same velocity to the penny and the paper.

166. **Galileo's Inclined Plane.**—To measure *how far* a body falls in each second is difficult, on account of the rapidity of the motion. Galileo used for this purpose an inclined plane, by which the motion was rendered slower. The plane was a very smooth plank, A B, with a groove along it, fixed at A, but having the end B movable up and down, so as to give the plane any required inclination. The body W, a smooth ivory ball, is allowed to run freely down the groove. It is acted upon by its own weight, W G. This may be resolved into W R, perpendicular to the plane, therefore ineffective, and W F moving W downwards. Now W G : W F : : A B : B C. Therefore, if B C be, suppose, $\frac{1}{8}$ of A B, the force acting on W will be $\frac{1}{8}$ of W G, *i.e.*, $\frac{1}{8}$ of gravity. Hence the space which would be traversed, if the ball fell freely, would be eight times the space actually traversed along the plane. In this particular instance, the distance traversed in one second, when measured, would be about 2 feet. Hence the space traversed, if gravity acted freely, would be about 16 feet. By many experiments, it was proved that the space traversed in 2 seconds would be 4 times 16; in 3 seconds 9 times 16, &c. That is, the spaces are proportional to the squares of the times. Also, the space traversed in the first second being 16, in the second second it is $16 \times 3 = 48$; in the third, $16 \times 5 = 80$; in the fourth, $16 \times 7 = 112$, and so on.

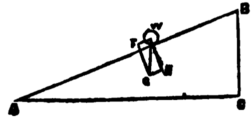


FIG. 71.

167. **Morin's Machine.**—A wooden cylinder, covered closely with paper, is made to rotate uniformly round a vertical axis by clock-work. A weight guided by two wires falls freely, and carries a pencil which marks a line on the paper. If the cylinder moved, and the weight did not, the pencil would mark a horizontal circle; if the weight fell, and the cylinder remained at rest, it would mark a vertical line; when both move, it marks a certain curve. When this curve

is completed, let the paper be cut along a vertical line, through the starting-point of the pencil, and unfolded. The curve is found to be of the form A e, Fig. 72 (a parabola). Let A B, B C, C D, D E, represent the spaces through which the cylinder turns in equal portions of time, say, tenths of a a second. Through B, C, D, E, draw vertical lines to meet the curve in b, c, d, e; these will show the space through which the weight falls in one, two, three, &c., tenths of a second. The lines B b, C c, D d, &c., will be as the numbers, 1, 4, 9, 16, &c., *i.e.*, as the squares of the times, as we found before. If A B, B C, &c., represent the space through which the cylinder turns in a second, B b will be 16 feet, C c = 16 × 4, D d = 16 × 9, &c.

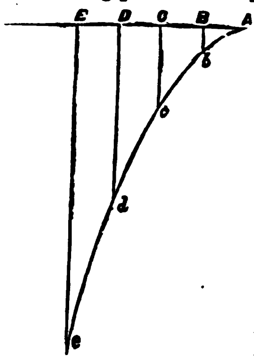


Fig. 72.

168. It has been assumed hitherto that gravity is a constant force. But it must be remembered that this is true only within narrow limits. *Outside the earth, its force varies inversely as the square of the distance, the body acted upon being the same.* If, for instance, a body weighs 100lbs. on the surface of the earth, *i.e.*, 4,000 miles from its centre, it should weigh only 25lbs. if removed 4,000 miles away from the surface of the earth. For if the distance be doubled, the attraction will be only ¼ of what it was. Hence, at the top of a mountain, the accelerative effect of gravity would be somewhat less than 32 feet per second. The mean force of gravity for the entire British Islands may be taken as 32·2 feet, or 9·8 metres, per second. For the sake of simplicity g will be taken = 32, in all the examples and exercises, unless when otherwise stated.

Within the earth (treating it as a homogeneous sphere or spheroid) the force of gravity is proportional to the distance from the centre.

Hence, outside the earth $g \propto \frac{1}{r^2}$

Within the earth $g \propto r$.

169. The laws of falling bodies may perhaps be more clearly understood from their graphic representation in Figs. 73, 74.

Let the vertical lines represent *time*, the horizontal lines

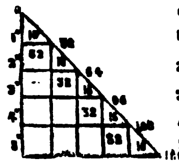


FIG. 73.

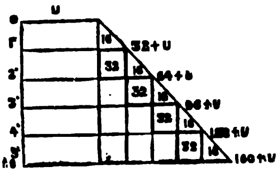


FIG. 74.

velocity, and the areas *spaces*. The dark square will represent the addition to the space passed over each second, and its upper side the addition to the velocity, or the acceleration.

170. In working questions on falling bodies, we may employ indifferently the equations of Art. 160 or of 164, substituting 32 or 32·2 for f or g.

Examples.

1. A stone falls to the bottom of a well in 4 seconds; what is the depth of the well?

$$s = \tfrac{1}{2} g t^2 = 16 \times 16 = 256 \text{ feet.}$$

2. A stone is thrown upwards with a velocity of 100 feet; when will it reach the ground?

Here we employ (1). To acquire a velocity of 100 feet per second, the stone must fall $\frac{u}{g}$ or $\frac{100}{32}$ seconds $= 3\frac{1}{8}$ seconds.

It will take $3\frac{1}{8}$ seconds in ascending, and the same time to return; therefore the whole time is $6\frac{1}{4}$ seconds.

3. After a body has fallen a hundred feet, what velocity will it have?

By (3) $v^2 = 64 \times 100 = 6400 \therefore v = 80$ feet.

4. If a body is projected downwards with a velocity of 5 feet per second, how far will it have descended in 7 seconds?

Here $s = ut + \frac{1}{2}gt^2 = 35 + 16 \times 49 = 819$ feet.

5. What will be the velocity of the same body at the end of 11 seconds?

The velocity due to projectile force will be 5 feet; that due to gravity will be $11 \times 32 = 352$ feet; in all, 357 feet.

6. From what height must a body fall to have a velocity of 320 feet?

$$v^2 = 2 g s \therefore 320^2 = 64 s \therefore s = 1600 \text{ feet.}$$

7. A body is projected vertically upwards, and moves for 8″ before it stops; what was its initial velocity?

Employing Art. 160, we select the equation in which s is not mentioned, viz., $v = u + f\,t$.

$v = 0$

u required.

$f = 32 \qquad \therefore o = u + 32 \times 8$

$t = 8 \qquad \therefore u = -256,$

the negative sign showing that the initial velocity was in the opposite direction to f – which was taken as $+ 32$ – and therefore upwards.

8. A stone is thrown upwards, and returns to the hand in 10″; with what velocity was it thrown?

v not mentioned — Select equation without v

u required

$f = 32 \qquad s = ut + \frac{ft^2}{2}$

$^*s = 0 \qquad \therefore 0 = 10\,u + \frac{32 \times 100}{2}$

$t = 10 \qquad \therefore -10\,u = 1600, \text{ or } u = -160 \text{ feet.}$

*171. We shall now consider the motion of bodies falling not freely, but **along a smooth inclined plane.** In Fig. 75 let W be a body moving down the plane A C. Draw Wc, representing the force of gravity acting on the body, and resolve it into W b perpendicular to the plane, and W a parallel to it. W a represents the force impelling W down the plane. Now $\frac{W\,a}{W\,c} = \sin. i \therefore W\,a = W\,c \sin. i = g \sin. i.$ Put $f = g \sin. i$ or $32 \sin. i$ in the formulæ, when solving, $v = u + f\,t$, &c. (Art. 160).

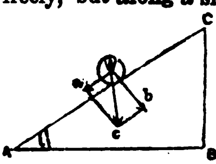


FIG. 75.

$\therefore$ (1) $v = gt \sin. i.$ (2) $s = \frac{1}{2}\,gt^2 \sin. i.$ (3) $v^2 = 2\,gs \sin. i.$

*172. *The velocity acquired by a body in running down a smooth inclined plane* is equal to the velocity acquired in falling freely through the height of the plane. For, let h = height C B, s = length A C. By (3) $v^2 = 2\,g\,s \sin. i = 2\,g\,h \therefore v = \sqrt{2\,g\,h}.$ But the velocity acquired in falling freely through h is also $= \sqrt{2\,g\,h}.$

* s is really the distance from the starting point.

173. *To find the time occupied in running down an inclined plane.*—Draw a circle BCDF (Fig. 76); make a vertical diameter BC. A body will run down any chord B D drawn from B in the same time as it will fall through the diameter B C. For, the time of running down B D is $= \sqrt{\frac{2\,s}{f}}$

But $s = \text{B D} = \text{B C} \sin. i$; and $f = g \sin. i$.

$\therefore t = \sqrt{\frac{2\ \text{B C} \sin. i}{g \sin. i}} = \sqrt{\frac{2\ \text{B C}}{g}}$; and

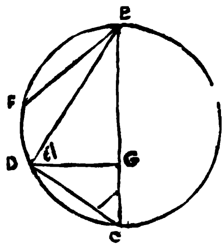


FIG. 76.

this is also the time of falling through B C. Now $\text{B C} = \frac{\text{B D}^2}{\text{B G}}$. Therefore the time occupied in running down an inclined plane will be the same as that occupied in falling through a height found by dividing the square of the length of the plane by its height.

EXERCISES.

1. Show that a body falling freely under the action of gravity from rest will in the fifth second describe a space nine times as great as the space described in the first second.

2. A body is thrown vertically downwards with a velocity of 10 feet per second; it reaches the ground in 12 seconds: what space has it described? *Answer.* 2,424 feet.

3. Describe a single experiment (without an air-pump) to show that all bodies would fall to the ground in the same time through the same height if it were not for the resistance of the air.

4. With what velocity must a heavy body be projected downwards that in 8″ it may overtake another which was dropped 3″ before it? *Answer.* 114·7 feet per second.

5. A stone dropped from a height reaches the ground in $2\frac{1}{2}$″. If now a stone be thrown downwards with a velocity of 3 feet per second, in what time will it reach the ground? *Answer.* 2·408″.

6. A weight of 10 lbs. is suspended from one end of a string; find the weight which must be attached to the other end, so that when the string passes over a fixed pulley the accelerating force may be half that of gravity. *Answer.* $3\frac{1}{3}$ lbs. or 30 lbs.

7. If a stone be thrown vertically upwards with a velocity of 200 feet per second, how long will it be before it reaches the ground again? *Answer.* 12·5″.

8. Show that a stone will fall from rest in 3″ through very nearly 150 feet; what velocity has it acquired? *Answer.* 96 feet per second.

9. If a body is thrown vertically upwards and moves for 6″ before it stops, what was the velocity with which it was thrown? *Answer.* 192 feet per second.

10. If a body be let drop from a height of 1,000 yards, how long will it be before it strikes the ground (disregarding the resistance of the air)? *Answer.* 13·65″.

11. Show that when a body falls from rest under the action of gravity, the space described varies as the square of the time from the beginning of motion.

12. If a stone is thrown upwards and returns to the hand in 8″, with what velocity was it thrown up? *Answer.* 128 feet per second.

13. A body under the action of a constant acceleration describes in three successive seconds distances of 12 feet, 18 feet, and 24 feet respectively, what ratio does the force producing the motion bear to the weight of the body? *Answer.* 3 : 16.

14. State exactly what is meant when it is said that the accelerating force of gravity near the earth's surface is very nearly 32·2 feet per second.

15. A body falls freely from rest under the action of gravity for 6″; what is the space described in the last 2″ of its motion? *Answer.* 320 feet.

16. If a body fell freely from rest, under the action of gravity ($g=32{\cdot}2$), for a quarter of a minute, show that it would then be moving at the rate of 483 feet per second, and ascertain what this velocity will be if expressed in miles per hour. *Answer.* 329·31 miles per hour.

17. Take A, B, C, three points in order in a vertical line, A being at the top: a body falls freely from A, under the action of gravity ($g=32$), and is observed to pass from B to C in 2″; if B is 144 feet above C, how many feet is A above B? *Answer.* 25 feet.

18. How long will it take a body to fall freely from rest through a vertical height of 192 yards? ($g=32$.) *Answer.* 6″.

19. A body falls from rest under the action of gravity ($g=32$) through 160 feet. How long will it take it to fall through the next 80 feet? *Answer.* ·711″.

20. A body is thrown upwards with a velocity of 96 feet per second; after how many seconds will it be moving *downwards* with a velocity of 40 feet per second (take $g=32$)? *Answer.* $4\frac{1}{4}$″.

21. The length of an inclined plane is 60 feet, its height 20 feet; what velocity would a body acquire in falling down that plane? *Answer.* 35·78 feet,

22. In what time will a body run down an inclined plane whose length is 50 feet, and height 10 feet? *Answer.* 3·9 seconds.

*23. If a cannon ball be fired vertically with an initial velocity of 2,400 feet, how far will it rise, and in how many seconds will it again reach the ground? *Answer.* 90,000 feet, 150 seconds.

24. A body projected vertically upwards against gravity has risen 120 feet in one second; what was its initial velocity of projection, and how far will it rise during the next second? *Answer.* 136 feet; 88 feet.

25. A stone is thrown vertically down a cliff 300 feet in height, and is observed to reach the base of the cliff in 4 seconds; what was the velocity of projection? *Answer.* 11 feet.

*26. Prove that the spaces described by a falling body in successive seconds are proportional to the odd numbers.

*27. Find the velocity with which a body should be projected down an inclined plane, so that the time of running down the plane shall be equal to the time of falling down the height.

28. If a body under an acceleration of 314 yards per minute acquire a velocity of 411 feet per second; find the space described. *Answer.* 5379·6 feet.

29. In what time will a bodv fall down a smooth plane 16 feet long inclined 60° to the horizon? *Answer.* 1·074 seconds.

CHAPTER IX.

LAWS OF MOTION.

174. The leading facts of dynamics, arrived at by experiment, have been summed up in three **laws of motion**, from which motions and the effects of forces can be deduced without further experiment. These were first clearly stated by Sir Isaac Newton, and are called **Newton's Laws.**

175. If a body be at rest, it will evidently not begin to move unless acted on by some force. If a force acts on a body for a single instant (*i.e.*, the smallest conceivable portion of time), the body will begin to move in the direction in which that force acts. Once in motion, if no force acts on it, there is nothing which either changes or destroys its motion. It will, therefore, continue to move in a straight line, and with uniform velocity. A force so acting is called an *impulsive*, or *instantaneous, force.* The principle just given, which is usually called the *Principle of Inertia,* is thus more fully expressed :—

Newton's First Law of Motion.—*A body will continue in its state of rest, or state of uniform motion in a straight line, unless compelled to alter that state by force impressed upon it.*

176. This law cannot be fully proved, since everything on the earth is acted on by gravity and friction, as well as by the impulsive force. But there can be no doubt of its truth. If I throw a bullet along a rough road it soon stops ; along a smooth surface it moves farther ; on ice farther still. Thus, the more the friction is lessened the farther the body moves. If a heavy top be spun in air, it stops in a few minutes ; if in a vacuum, it rotates for a much longer time. The same is the case with a pendulum ; in air it stops after a short time ; *in vacuo,* it may be made to oscillate for hours. Again, it is found that a constant expenditure of fuel maintains a uniform speed in a railway train, after it has once got up to the required speed. The fuel consumed just balances the friction and other resistances, and the train moves uniformly, since all the forces acting on it are now in equilibrium. If friction, and all other opposing forces, such as the attraction of the earth and resistance of the air, could be removed entirely, no doubt these bodies would continue to move for

ever at the same rate. Hence, an impulsive force imparts a *uniform velocity*.

177. If a force not merely sets a body in motion, but continues to act on it, it is called a *constant*, or *continuous*, force. Acting for an instant it would cause the body to start into motion with a certain uniform velocity; but as it acts equally during the next instant, it will impart an additional velocity equal to the first; and every successive instant the velocity will receive an equal increase. Thus, if a force acting on a body for one second imparts a velocity of 5 feet per second, and if it continues to act, the velocity at the end of the next second will be 10 feet, at the end of the third second 15 feet, and so on. If we call the velocity which the force imparts by acting for one second f, the number of seconds t, and the velocity at the end of any given time v, we shall have $v = ft$.

The amount, f, added to the velocity in each unit of time is called the **acceleration**, or *accelerative effect*, of the force, and sometimes the *accelerating force*. In dynamics forces are measured by their acceleration, that is, by the velocity which they generate in each unit of time in some known mass.

178. The **volume** of a body means the space which it occupies, estimated in cubic inches, feet, or yards.

The **mass** of a body means the quantity of matter it contains. The quantity of matter in any particular volume depends on the closeness with which the particles are packed together. Distilled water at a temperature of 62° F. is employed as a standard for comparison of masses. The **density** of a body is the ratio of its mass to that of an equal volume of the standard. At the same place on the earth's surface the mass of bodies is proportional to their weight, and hence masses may always in practice be estimated by weights. But it must be remembered that the mass of any body remains constant, while its weight may vary, according to its position on the earth's surface or its elevation above the surface. For instance, a cubic inch of iron weighs 7·248 times as much as a cubic inch of water; the density of iron is, therefore, 7·248. A cubic inch of copper weighs 9 times as much as a cubic inch of water; therefore the density of copper is 9. When the weight of any portion whatever of a body is proportional to the volume of that portion, the body is of *uniform density*, or is *homogeneous*.

179. In considering questions respecting the motion of bodies, it is necessary to take into account both the mass in motion and the velocity with which it moves. The **momentum** of a moving body, or its **quantity of motion**, is the product of its mass by its velocity. Two bodies of different masses may have the same momentum. Thus, a cannon ball weighing 24lbs. and moving with a velocity of 20 feet per second will have a momentum of 480. A ball weighing ¼lb. and moving with a velocity of 1,920 feet per second will have the same momentum; though, as we shall see hereafter, their effects will be very different. Momentum $= Mu$ at time t: change of momentum during time $t = Mv - Mu = M(v-u) = Mft$ by Art. 160, equation (4) $\therefore$ change of momentum (or momentum generated) in a unit of time $= \dfrac{Mv - Mu}{t} = Mf$.

180. **Second Law of Motion.**—*When a force acts on a body in motion, the change produced in the quantity of motion is proportional to the force applied, and takes place in the direction of that force.*

Sitting in a railway carriage at rest, I throw up a heavy ball with force just sufficient to make it touch the top of the carriage. The train starts, and, when it is moving 30 miles an hour, I throw up the ball again with the same force; it just touches the top of the carriage as before and drops at the same spot. So that the force with which I act on the ball when it is moving 30 miles an hour has the same effect as it had on the ball at rest. A rider at full speed throws a heavy ball straight up; to him it appears to rise straight up, and fall straight down again into his hand; to a spectator at rest it has a very different motion. A sailor at the top of a mast drops a marble to the deck; where will it fall? Close beside the mast; its two motions being compounded, it moves downward by the force of gravity, but it retains also the forward motion which it had in common with everything else in the ship. The movements of the heavenly bodies are calculated in accordance with this law, and afford a striking confirmation of it.

181. Let f be the acceleration produced by a force of F lbs. acting on a body of mass M; then if another force of F lbs. be put to act on the same body in the same direction, the whole acceleration will be $2f$: for experience shows that each force F will produce the same effect as if it were acting

alone $\therefore$ each force will add f feet per second to the existing velocity, every second $\therefore$ the total addition to the velocity every second will be $2f$ feet per second. By extending this mode of reasoning, it may be shown that *as long as we keep the same mass the acceleration will be directly proportional to the statical force which causes it;*

$$i.e., f \propto \text{F when M is constant} \quad . \quad . \quad \text{(i)}.$$

Again, let $\frac{F}{3}$ lbs. act on mass M: the acceleration will be $\frac{f}{3}$ by the preceding paragraph. Now take three bodies of mass M, each acted on separately by a force $\frac{F}{3}$ lbs., each will have an acceleration of $\frac{f}{3}$. Hence we find a total force of F lbs. acting on a total mass of 3 M produces an acceleration $\frac{f}{3}$ throughout it; *i.e.*, keeping the same force F, but taking three times the mass, the acceleration becomes one-third; by extending this mode of reasoning it may be shown that *as long as the statical force is constant, the acceleration is inversely proportional to the mass moved;* or

$$. \quad . \quad . \quad f \propto \frac{1}{M} \text{ when F is constant} \quad . \quad . \quad \text{(ii)}.$$

Again, if m F lbs. act on mass M, the acceleration is mf by (i). Now keep the same force, but change the mass to n M; then by (ii) the acceleration will be changed into $\frac{mf}{n}$; thus when $\frac{F}{M}$ is changed into $\frac{mF}{nM}$ or $\frac{m}{n} \times \frac{F}{M}$, f is in consequence changed into $\frac{mf}{n}$ or $f \times \frac{m}{n}$; *i.e., the value of the acceleration is changed in the same proportion as the fraction* $\frac{F}{M}$ $\therefore$ when both F and M vary $f \propto \frac{F}{M}$ . . (iii).

Now, if the force acting on M were its own weight, the resulting acceleration would be g (= 32 feet per second).

$\therefore$ When $F = W, f = g$, M being the same, thus the variation of f is $\frac{f}{g}$, when that of $\frac{F}{M}$ is $\frac{F}{M} \div \frac{W}{M}$ $\therefore$ by (iii)

$$\frac{f}{g} = \frac{F}{M} \div \frac{W}{M} = \frac{F}{W}, \text{ or } \quad . \quad . \quad . \quad F = \frac{W}{g} \times f \quad . \quad . \quad . \quad \text{(iv)}.$$

Or more generally thus: If it be found in a special case that

f_o F_o M_o are the values, then in every other case the ratio of f to f_o must be = that of $\frac{F}{M}$ to $\frac{F_o}{M_o}$ $\left(\because \text{by (iii)} f \propto \frac{F}{M}\right)$

$\therefore \frac{f}{f_o} = \frac{F}{M} \div \frac{F_o}{M_o}$ . . . whence $F = \frac{F_o}{M_o f_o} \times M f$ = a constant $\times$ Mf; or, for shortness, say

$$F = c\,M f \quad . \quad . \quad . \quad \text{(v)}.$$

182. Now, we can determine c experimentally, and we shall find that its value depends on the units we employ in the measurement of the three quantities, for example :—

(a) Let the unit of mass be the mass of a cubic foot of water, the unit of force 1lb., and of acceleration 1 foot per second; and apply equation (v) to the case of a cubic foot of water acted on by its own weight.

$$\text{Here } F = 1000 \text{ oz.} = \frac{1000}{16} \text{ lbs.}$$

$$M = 1$$

$$f = 32 \text{ feet per second,}$$

$$\text{and thus } \frac{1000}{16} = c \times 1 \times 32$$

$$\therefore c = \frac{125}{64}.$$

(b) Or, if we take 1 cubic inch of water as unit of mass,

$$\frac{1000}{16 \times 1728} = c \times 1 \times 32 \therefore c = \frac{125}{110592}.$$

(c) If, again, we take $\frac{64}{125}$ of a cubic foot of water as unit of mass, we have

$$\text{Its weight} = 32\text{lbs., and by (v)}$$

$$32 = c \times 1 \times 32, \text{ or } c = 1.$$

Thus equation (v) becomes $F = Mf$ . . . (vi). When we use the equation in this form the unit of mass is that quantity of matter in which 1lb. creates an acceleration of 1 foot per second. But, by Art. 179, Mf is the momentum generated in a unit of time: hence we see that a force is measured dynamically by the momentum it creates in a unit of time.

NOTE.—Equation (iv) may be obtained thus :—In all cases $F = Mf$ $\therefore$ in the special case when the body falls under the action of its weight—

$$W = Mg \text{ or } M = \frac{W}{g} \therefore F = Mf = \frac{W}{g} f.$$

183. Consider the case of two unequal bodies, P and Q, connected by a cord which passes over a fixed pulley.

Neglecting friction, the tension of the cord will be the same throughout: let T be the tension in each part.

Consider the motion of Q, the less body. The force T causes it to move, but is partly balanced by its weight Q $\therefore$ the actual moving force is T − Q, and the weight moved by it is Q $\therefore$ using $F = \frac{W}{g} f$, we obtain

$$T - Q = \frac{Q}{g} f \quad . \quad . \quad . \quad \text{(i)}$$

In the case of P, we get, in like manner, as the weight is greater than T—

$$P - T = \frac{P}{g} f \quad . \quad . \quad . \quad \text{(ii)}$$

The f is the same in the two cases, because the thread is inextensible; $\therefore$ P and Q must always have the same velocity, and the same increment of velocity. Adding, we get

$$P - Q = \frac{P + Q}{g} \cdot f \therefore f = \frac{P - Q}{P + Q} g \quad . \quad . \quad \text{(iii)}$$

Dividing (i) by (ii) we get

$$\frac{T - Q}{P - T} = \frac{Q}{P}, \text{ whence } T = \frac{2\,P\,Q}{P + Q} \quad . \quad . \quad . \quad \text{(iv)}$$

184. The unit usually employed to measure dynamical force is the **Absolute Unit of Force**, and is defined as "*that force which, acting for unit time, would impart unit velocity to unit mass.*" It is also frequently called the *Gaussian unit*. Taking 1lb. as unit mass, 1 second as unit time, and 1 foot per second as unit velocity, we can determinethe unit of force. A pound of matter falling freely by its own w eight would, like any other body in similar circumstances, acquire in one second a velocity of 32·2 feet per second. The weight which would impart to it a velocity of 1 foot per second is therefore $\frac{1}{32 \cdot 2}$lb. or $\frac{16}{32 \cdot 2}$oz. (almost exactly half an ounce) or $\frac{7000}{32 \cdot 2}$grs. = 217·4grs. A pressure of $\frac{1}{32 \cdot 2}$lb. or 217·4grs., therefore, acting on a body free to move, is the absolute unit of force. The name **Poundal** is now generally given to this unit. It is $= \frac{1}{g}$lb.

185. In the Metric or French system, the absolute unit is the force which, acting on 1 gramme for 1 second, would impart a velocity of 1 centimetre per second. The gramme falling

freely would acquire in 1 second a velocity of 981 centimetres; therefore to produce a velocity of 1 centimetre, a force of $\frac{1}{981}$ gramme would be required. The *metric absolute unit*, or **Dyne**, therefore is $\frac{1}{981}$ gramme.

186. **Third Law of Motion.**—*Action and reaction are equal and opposite; or, if two bodies mutually act upon each other, the quantities of motion developed in each in the same time are equal and opposite.*

If I press my hand on the table, the table presses against my hand with an equal force. When a horse pulls a carriage forward, the carriage equally pulls the horse backward. If one body strikes another and changes its motion, its own motion undergoes an equal change in the opposite direction. When a bullet is fired from a gun with a certain velocity, the quantity of motion imparted to it is its mass × its velocity. An equal momentum is imparted to the gun, which recoils or *kicks*. Since the mass of a body is proportional to its weight, we may in comparing the momenta, employ the weights of the bodies instead of their masses. Hence, the weight of the gun × velocity of recoil = weight of bullet × velocity of projection. The same is the case in attraction of various kinds. A magnet attracts a piece of iron; the iron equally attracts the magnet. The earth attracts a falling body; the falling body equally attracts the earth, which moves to meet it, though its motion is so slight as to be insensible. Weight of the earth × its velocity = weight of body × its velocity. This law is of great importance in Practical Mechanics.

FIG. 77.

187. **Attwood's Machine** (Fig. 77) is an instrument which measures, accurately and beautifully, the accelerative effect of gravity. Over a pulley, A, passes a thread bearing two equal and similar weights, P, Q. These are, of course, in equilibrium. Let a small weight p, be placed on P, it will cause motion. The pressure is p, the weight moved is $P+Q+p=2P+p$. The accelerating force is, therefore, that fraction of gravity denoted by the fraction $\frac{p}{2P+p}$. If, for example, $2P+p=16p$, the moving system is acted on by only $\frac{1}{16}$ of gravity. Let the

weight P start from the position in which its upper surface is on a level with s, marked on the scale, and let it move for *one* second: mark the place s_1, where it arrives in 1″. Then, starting it again at s, let it move for *two* seconds, and let it then arrive at s_2. The distance $s\,s_2$ will be found equal to $4\,s\,s_1$. Let now the motion continue *three* seconds, and the space $s\,s_3$ will be $= 9\,s\,s_1$; in *four* seconds it will be $=16\,s\,s_1$. Hence it is shown that *the space described is proportional to the square of the time.*

188. Again, when P has moved for one second, let the ring, r (which is movable up and down), be placed so as to remove p from P at the end of 1″. P will continue to move downward with the same velocity which it had at s_1. Let now the stage, m, be so placed as to receive P exactly 1″ after passing r. Then $r\,m$ will indicate the velocity acquired in *one* second. If P be allowed to move for *two* seconds before passing r, and m receive it as before, at the end of another second, $r\,m$ will then indicate the velocity acquired in two seconds. This will be twice as great as before. In the same way the velocity acquired in falling *three* seconds will be three times as great; in *four* seconds, four times, &c. Hence, *the velocity acquired is proportional to the time.*

If P and Q be each $15\frac{1}{2}$oz., and p 1oz., $\frac{1}{32}$ of gravity will be effective, and it will be found that the space traversed in the first second will be about six inches; the velocity at the end of the first second, one foot per second. Hence, if the whole weight were effective, these numbers would be about 16 and 32 feet respectively. More accurately, 16·1 and 32·2 feet.

189. The friction of the pulley A is lessened as much as possible by resting each end of its axis on two overlapping wheels turning freely: these are called *friction wheels.*

Besides friction, there is another source of error in the calculations, namely, that the moving force p has not only to move the mass of $2\,P + p$, but also to move the pulley, or overcome its inertia.

EXERCISES.

1. A body whose mass is 8lbs. is known to be under the action of a single constant force. It is observed to move from rest, and in the first second of its motion to describe a distance of 5 feet. What is the magnitude of the constant force? *Answer.* $2\frac{1}{2}$lbs.

2. Required the statical force which acting freely for a second on the mass of a ton weight will get up in it a velocity of one foot per second. *Answer.* 69·56lbs.

*3. A weight of 10lbs. drags another weight of 8lbs., the weights being connected by a string which passes over a fixed pulley, and both weights hanging vertically; with what acceleration does the centre of gravity of the weights descend? *Answer.* The acceleration of the weights is $\frac{1}{9}$ of g; the centre of gravity will descend at the rate of $\frac{1}{9}$ of this acceleration, or $\frac{1}{81}$ of $g = \frac{32}{81}$ feet per second.

4. A body containing 50lbs. of matter is set in motion by a constant force which acts for 5 seconds in the direction of the motion; it then ceases to act, and the body (now acted upon by no external force) moves over 60 feet in the next two seconds. Find (1) the accelerative effect of the force, (2) the magnitude of the force in absolute units. *Answer.* Six feet per second; 300 absolute units.

5. Two masses of 48 and 50 grammes respectively are attached to the string of an Attwood's machine, and, starting from rest, the larger mass passes through 10 centimetres in one second. Determine from these data the value of the acceleration due to gravity, your units being centimetres and seconds. *Answer.* 980.

6. What is meant by saying, with reference to gravity, $g = 32$? What would be the value of g if your units of space and time were miles and hours? *Answer.* 78545·45.

7. A weight of 300 lbs. resting on a plane of 60° inclination, draws (by means of a string passing over a pulley at the common vertex of the two planes) another weight of 200 lbs. resting on a plane of 30° inclination; find the tension of the string. *Answer.* $60(\sqrt{3}+1)$.

8. A falling body, P, draws another body, Q, by a connecting cord, along a smooth horizontal plane: required the tension on the cord. *Answer.* $\frac{PQ}{P+Q}$.

CHAPTER X.

PROJECTILES.

190. If a body were projected vertically upwards with velocity u, it would, if not acted upon by gravity, rise in t seconds to a height $= ut$; but in the same time gravity would cause it to fall, or, what amounts to the same thing, retard its ascent, through a space $= \frac{1}{2} gt^2$. Therefore its actual ascent in time t would be $s = ut - \frac{1}{2} gt^2$. If the body were projected vertically downwards, the space passed over in time t would be $s = ut + \frac{1}{2} gt^2$.

191. Let us consider now the case of a body projected obliquely near the earth. Such a body is called a **projectile**, and, in treating of its motion, we regard it as a heavy particle moving in a vacuum, *i.e.*, we omit the size of the body and the resistance of the air. Suppose a body projected from A along the line A B (Fig. 78) with velocity u, we know that it will move along a curved line, called its *trajectory*, and reach the ground at some point C. The line A B is called the *line of projection*, the angle e is the *elevation*, and the distance A C is the *range* of the projectile.

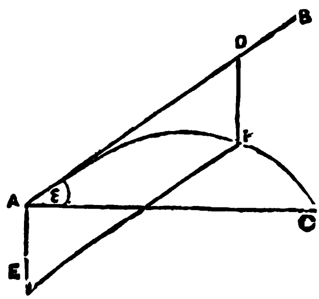


FIG. 78.

The body is evidently acted on by two forces, an impulsive force, which sets it in motion along the line A B, and the force of gravity, which is a constant force acting vertically downwards. If gravity did not act, the body would move uniformly along A B; let D be the point at which it would arrive in time t $\therefore$ A D $= ut$. But under the action of gravity it would in the same time fall through a space $= \frac{1}{2} gt^2$, represented by A E. Therefore under the action of both forces, the projectile will at the end of the time t be found at F. $DF = AE = \frac{1}{2} gt^2$; $AD = EF = ut \therefore EF^2 = u^2 t^2$; and $t^2 = \frac{2\,AE}{g} \therefore EF^2 = \frac{2\,u^2\,AE}{g}$; hence $\frac{(EF)^2}{AE} = \frac{2\,u^2}{g}$; and this value is always the same, whatever be the time taken. Therefore the curve is such that the *abscissa* A E varies as the

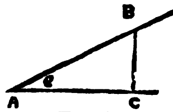


FIG. 79.

square of the *ordinate* E F, *i.e.*, *the path of a projectile is a parabola.*

*192. If we take A B (Fig. 79) as representing the velocity of projection u, we can resolve it into a horizontal component A C, and a vertical component C B. Now, $\frac{AC}{AB} = \cos. e \therefore AC = AB \cos. e = u \cos. e$; and $\frac{CB}{AB} = \sin. e \therefore CB = AB \sin. e = u \sin. e.$

The horizontal component remains unchanged during the whole time of flight, since gravity acts at right angles to it, and therefore does not affect it. But the vertical component is partly counteracted by gravity.

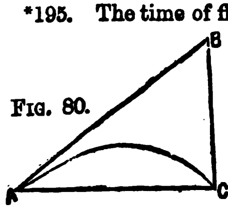
To find the greatest height to which a projectile will rise.—It will rise to the same height as if projected vertically with velocity $= u \sin. e$. Now $u^2 = 2gs \therefore s = \frac{u^2}{2g}$. For u substitute $u \sin. e$; therefore $s = \frac{u^2 \sin.^2 e}{2g}$.

*193. *To find the time of flight.*—The time of the ascending path is that due to velocity $u \sin. e$, *i.e.* $= \frac{u \sin. e}{g}$, and the time of descent will be the same; therefore the whole time of flight $= \frac{2u \sin. e}{g} = \frac{u \sin. e}{16}$.

*194. *To find the range on a horizontal plane.*—The time of flight is $\frac{2u \sin e}{g}$, and the horizontal velocity is $u \cos. e$; therefore the range $= \frac{2u^2 \sin. e \cos. e}{g} = \frac{u^2 \sin. 2e}{g}$.
The greatest value of $\sin. 2e$ is 1, which corresponds to $e = 45°$; therefore the range is greatest at elevation 45°.

*195. The time of flight and range may also be found thus:

FIG. 80.



$AB = ut$, $BC = \frac{1}{2}gt^2$

$\text{Sin. } e = \frac{BC}{AB} = \frac{\frac{1}{2}gt^2}{ut} = \frac{gt}{2u} \therefore t = \frac{2u \sin. e}{g}$.

$\frac{AC}{AB} = \cos. e \therefore AC = AB \cos. e = ut \cos. e = \frac{2u^2 \sin. e \cos. e}{g} = \frac{u^2 \sin. 2e}{g}$

*196. **To find the position, velocity, and direction at any time t.**

Let v be the velocity, and θ the inclination to the horizon at time t. Then—

Vertical component of velocity $= v \sin. \theta$

Horizontal component of velocity $= u \cos. \theta$.

$\therefore$ Applying the general formula, $v = u + ft$ to the vertical motion only—

$$v \sin. \theta = u \sin. e - gt \quad . . \text{(i)}$$

$$\text{Also } v \cos. \theta = u \cos. e \quad . . . \text{(ii)}$$

Because the horizontal component remains unchanged.

$$\therefore v^2 = v^2 \cos.^2\theta + v^2 \sin.^2\theta$$
$$= u^2 \cos.^2 e + u^2 \sin.^2 e - 2\, ugt \sin. e + g^2 t^2.$$
$$= u^2 - 2\, ugt \sin. e + g^2 t^2 \text{ (iii)}$$
$$\text{and } \tan. \theta = \frac{v \sin. \theta}{v \cos. \theta} = \frac{u \sin. e - gt}{u \cos. e}. \text{ (iv)}$$

(iii) and (iv) give the velocity and direction at any given time t.

197. The parabolic theory is almost useless in practice, since the resistance of the air, the rotation of the projectile, and other causes interfere with the motion. But it is useful as an approximation to the actual path, and it affords a good example of the application of mechanical principles. The actual range is usually only a small fraction of the theoretical.

EXERCISES.

1. Find the horizontal range and time of flight of a ball whose velocity is 400 feet, elevation 45°. *Answer.* 5000 feet; 17·67 seconds.
2. Find the range and time of flight of a projectile whose velocity is 400 feet, elevation 30°. *Answer.* 4330·125 feet; 12½ seconds.
3. Find the range and time of flight of a projectile whose velocity is 400 feet, elevation 60°. *Answer.* 4330·125 feet; 21·65 seconds.
4. A body is projected with a velocity of 600 feet per second, at an elevation of 30°; find the greatest height to which it will rise. *Answer.* 1406·25 feet.
5. With what velocity should a body be projected at an elevation of 60° to have a range of 1 mile? *Answer.* 441·7 feet.
6. At what elevation should a body be projected with a velocity of 1000 feet per second to have a flight of 31¼ seconds? *Answer.* 30°

CHAPTER XI.

CIRCULAR MOTION AND THE PENDULUM.

198. A particle in motion, not acted on by any force, would, as we have seen, continue *to move uniformly in a straight line.* If a force acts on the particle, it must change either the velocity or direction of its motion, or both. If it acts along the same line as the particle is moving in, it will change the velocity only, either accelerating or retarding it; but if it acts in a different line it will change the direction of motion, and may or may not alter the velocity. In the latter case the particle will, under certain circumstances, move with uniform velocity in a circle. We must now investigate the conditions under which such motion will take place.

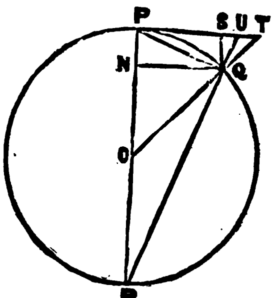


FIG. 81.

*199. A particle of weight W describes the circumference of a circle of radius r with uniform velocity v; it is required to find the force acting upon it at any time (Fig. 81).

Let F be the force, and f its accelerating effect on the particle. Let P be the position of the particle at any instant, and let it describe the arc P Q in t'' so that, as v is the distance it describes in 1'',

Arc P Q $= v\,t$.

The velocity v at P is in the direction of the tangent, and receives no increase, $\therefore$ there can be no acceleration in the direction of the tangent, and $\therefore$ the whole acceleration is at right angles to the tangent in the direction P R; hence the diameter P R is the direction of the force F; and as the same happens at every point (viz., there is no acceleration in direction of the tangent, and the line at right angles to the tangent is the diameter) we see that F always acts towards the centre O.

If we now cut off P U on the tangent = arc P Q, then the particle would have been at U if no force had acted, and the effect of F has been to bring it to Q instead of U, or Q U is the displacement due to F in time t.

Draw Q N and Q S at right angles to P R and P U respectively, and join P Q and Q R.

Let us now suppose Q brought indefinitely near to P, the points N S U T being obtained in its new position; the time t will then be indefinitely small, but we shall still have—arc $PQ = vt$. The points S U T will become indefinitely near to each other, so that Q S in its new position may be taken instead of Q U as the distance through which the particle is displaced in time t with an acceleration f.

In the case of the large arc P Q the acceleration evidently varies in direction as the particle passes along it, and we have as yet no proof that it does not vary in magnitude; but in the indefinitely small arc P Q there is not room for variation in either respect; so that we may use the formula $s = \frac{ft^2}{2}$ (which only applies to a constant acceleration in a constant direction), and say $QS = PN = \frac{1}{2} ft^2$.

Now, by the equiangular triangles P N Q and P Q R, we have $\frac{PN}{PQ} = \frac{PQ}{PR}$ or $PN. PR = PQ^2 = (\text{arc } PQ)^2$ when the arc is indefinitely small—

$$\therefore \tfrac{1}{2} ft^2 \times 2r = (vt)^2 = v^2 t^2$$

$$\therefore fr = v^2, \text{ and } f = \frac{v^2}{r}$$

a result independent of the position of P. Thus we see that f is constant, and $F = \frac{W}{g} f = \frac{Wv^2}{gr}$

Now, Let T be the periodic time, or the time of a complete revolution; then vT = whole circumference $= 2\pi r$

$$\therefore f = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$\text{and } F = \frac{W}{g} f = \frac{4W\pi^2 r}{gT^2}$$

200. Hence, uniform motion in a straight line is changed into uniform motion in a circle, under the action of a central force which changes the direction, but does not affect the velocity of motion. The force F, which acting on the particle compels it to leave its straight path and describe the circumference of a circle, is called the *centripetal force*, as acting towards the centre. It exactly balances the tendency of the particle to continue its motion in a straight line along the tangent; in other words, its tendency to fly away from the centre. The

force with which the body tends to fly away from the centre is called the *centrifugal force.* Since it balances the centripetal force, it is equal and opposite to it; *i.e.*, it acts from the centre, and is $= F = \dfrac{Wv^2}{gr} = 4W\dfrac{\pi^2 r}{gT^2}$.

Hence it is directly proportional to the square of the velocity, and inversely proportional to the radius of the circle; or, *it is directly proportional to the radius, and inversely proportional to the square of the periodic time.*

Examples.

1. A stone 1lb. in weight is tied to a string 3ft. long and whirled round with a velocity of 24ft. per second; find the strain on the string (or the centrifugal force).

$$\text{Here } F = \frac{Wv^2}{gr} = \frac{1 \times 24 \times 24}{32 \times 3} = 6\text{lbs. Ans.}$$

2. A body weighing 12lbs. describes a circle of radius 3 yards in $2\frac{3}{4}$ seconds with uniform velocity; find the centrifugal force.

Here
$$F = \frac{4W\pi^2 r}{gT^2} = \frac{4 \times 12 \times \pi^2 \times 9}{32 \times 2\frac{3}{4} \times 2\frac{3}{4}}$$
$$= \frac{27 \times 9\cdot86965}{2 \times 7\cdot5625}$$
$$= 17\cdot6185\text{lbs. } \textit{Ans.}$$

or,
$$vT = 2\pi r$$
$$\therefore v \times 2\tfrac{3}{4} = 2 \times 3\cdot1416 \times 9$$
$$\therefore v = \frac{8 \times 9 \times 3\cdot1416}{11}$$
$$= 20\cdot5632 \text{ ft. per sec.}$$
$$\therefore F = \frac{Wv^2}{gr} = \frac{12 \times (20\cdot5632)^2}{32 \times 9}$$
$$= 12 \times \cdot0714 \times 20\cdot5632$$
$$= 17\cdot6185\text{lbs. } \textit{Ans.}$$

201. **Examples of Centrifugal Force.**—When a weight attached to one end of a cord, the other end of which is held in the hand, is swung round in a circle, the centrifugal force generated causes a considerable tension in the string. If this tension be greater than the cohesive strength of the string, which may be regarded as the centripetal force, the string will break, and the weight will be projected away from the hand.

The form of the earth strikingly illustrates the effects of

centrifugal force. The portions where the rotatory motion is greatest possess the greatest centrifugal force, and the earth is therefore swelled out at the equator, and has the form of a flattened spheroid. At the equator the centrifugal force is about $\frac{1}{289}$ of the weight of bodies. If the earth moved seventeen times as fast, bodies there would have no weight; if faster still, they would be projected off the earth.*

In running round a circle a person must, in order to counteract centrifugal force, lean inwards. So the outer rails on a railway curve are elevated to incline the carriages inward, and thus counteract the centrifugal force which urges them outwards.

202. **The Pendulum** is an instrument of very extensive use in mechanics. It is used to measure time, to determine the intensity of gravity at different places, and to prove the precise figure of the earth. *Any body suspended by a fixed horizontal axis, and capable of moving about that axis,* is a pendulum. Such a body will remain at rest only when its centre of gravity is directly below the axis of suspension. If moved from this position it will oscillate backwards and forwards for some time, and finally come to rest in the same position.

203. **The Simple Pendulum** consists of a *heavy particle* suspended by an inextensible and *perfectly flexible* thread *without weight.* If raised to B (Fig. 82), and let go, the particle tends to fall downwards with a force, B G, equal to its weight. This can be resolved into B F, which is destroyed by the resistance of the thread C B, and B H, which causes it to move towards A. In descending from B to A, it acquires a velocity sufficient to raise it to D. This velocity is constantly retarded by the action of gravity: at D it disappears, and the body then descends as before to A. It gains in each descent precisely as much motion as it lost in the ascent; hence if the resistance of the air and the friction of the thread were destroyed the

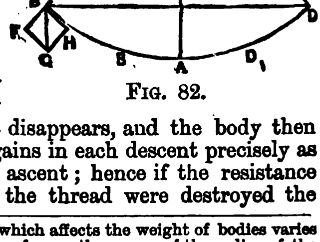


FIG. 82.

* The portion of centrifugal force which affects the weight of bodies varies as the square of the velocity, and therefore as the square of the radius of the circle described; and therefore, finally, as the square of the cosine of the latitude.

motion would continue for ever. In the exhausted receiver of an air-pump, the pendulum moves for a much longer time than in air.

The motion of the pendulum from B to D, or from D to B, is called a *vibration* or *oscillation* (*oscillum*, a little wing, L). The arc, B D, is called the *amplitude* of the vibration. The motion from B to D, and back from D to B, is a *double vibration*, and the time occupied by it a *complete period.*

204. **Isochronism** (*isos*, equal; *chronos*, time, Gr.). For small arcs the time of vibration is independent of the amplitude. That is, a pendulum always vibrates in the same time, whether it vibrate from B to D, or from B_1 to D_1, provided its length remains the same. Hence the use of the pendulum in clocks to mark equal portions of time.

205. **The time of a single small oscillation** is experimentally shown to be the time of falling through half the length of the pendulum $\times$ 3·1416. $s=\frac{1}{2}gt^2 \therefore t=\sqrt{\frac{2s}{g}}$. Now, $2s = l \therefore t = \sqrt{\frac{l}{g}}$, is the time of falling through half the length. Therefore, the time of a single small vibration is given by the formula,

$$T = \pi\sqrt{\frac{l}{g}} \ldots (\pi = 3{\cdot}1416 \ldots g = 32{\cdot}2)$$
$$\text{or } T^2 g = \pi^2 l.$$

Hence *the time of vibration is proportional to the square root of the length*, while the force of gravity remains constant. If the pendulum be made four times as long the time of vibration is doubled. It follows from this that the square of the number of vibrations is inversely proportional to the length.

$$T : t :: \sqrt{L} : \sqrt{l}, \quad \text{or } T^2 : t^2 :: L : l.$$
$$N^2 : n^2 :: l : L; \quad \text{or } N : n :: \sqrt{l} : \sqrt{L}$$

206. **Compound Pendulum.**—Simple pendulums have no real existence. Our ordinary pendulums are compound—that is, where the thread is not weightless, and the *bob* is not a mere particle, but a body of sensible size. The law of isochronism holds good for compound pendulums. This was discovered by Galileo in 1582. Observing a heavy lamp hanging from the roof of the Cathedral of Pisa, and set in motion, he found, by comparison with his own pulse, that it made all its vibrations in the same time, though they were not of the same amplitude. By employing balls suspended

by threads of different lengths, he ascertained that the time of vibration varies as the square root of the length. A pendulum vibrating seconds at London is 39·1393 inches long; therefore a pendulum which would vibrate in 2 seconds must be 39·1393 inches × 4, and so on. This rule holds while the pendulum consists of a small ball at the end of a long string, *i.e.*, as nearly as possible a simple pendulum. When it is of a different form the *length* has to be found by experiment.

207. Suppose while a compound pendulum is vibrating it were suddenly to break up into its separate particles, and that these particles should continue to vibrate as before round the centre of suspension. We have thus a great number of simple pendulums vibrating round the same point. They will vibrate in very different times. Those near the point of suspension will vibrate more quickly than before; those furthest from it more slowly. The intermediate particles will vibrate at all intermediate rates. Among these there must be some one rate of vibration which corresponds with that of the whole body. The point which vibrates at the same rate as the compound pendulum is its *centre of oscillation*. If every particle could be pressed into this spot the simple pendulum so formed would oscillate in the same time as the compound pendulum does. It is called the **equivalent simple pendulum.**

208. The **length** of a compound pendulum, then, means the length of a simple pendulum, which would vibrate in the same time. It is equal to the straight line joining the point of suspension with the centre of oscillation. We shall now show how to determine this length.

209. **Convertibility of the Centres of Oscillation and Suspension.**—It was discovered by Huyghens that if a pendulum, A B (Fig. 83), be suspended by O, its centre of oscillation, it will continue to vibrate in the same time as before. Hence, if O be made the centre of suspension, S, the former point of suspension, becomes the centre of oscillation. To find the length of the pendulum A B, then, suspend it first at S, and observe carefully the number of vibrations in a minute. Then find by trial some other point O—such that when the pendulum is suspended from it the number of vibrations is the same as before. The distance S O is the length required. A compound pendulum of uniform thickness and density will vibrate in the same time as a simple pendulum of $\frac{2}{3}$ its length, provided it be suspended from one extremity.

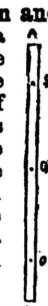


FIG. 83.

210. **Kater's Pendulum** can vibrate about either of two parallel axes, one of which (suppose at S, Fig. 83) is fixed, while the other, O, can be moved along the pendulum. It is first suspended at S, and the number of vibrations per minute observed. It is then suspended from O, which is moved up or down, until after a few trials it is found to be so placed that the rate of vibration is the same as before. The distance S O is then the length of the equivalent simple pendulum.

211. **Determination of the Force of Gravity.**—From the formula $T = \pi\sqrt{\frac{l}{g}}$, we get $g = \frac{\pi^2 l}{T^2}$. The force of gravity at any place can therefore be found by making any pendulum vibrate, and finding the values of T and l. T is found by counting the number of vibrations in a given time; l is found as shown in the preceding paragraphs. If T be taken as 1″, we have for the force of gravity at any place, $g = \pi^2 l$. If l be taken in inches, of course g will be found in inches also.

212. **Inequality of the Force of Gravity at Different Places.**—Pendulum experiments show that the force of gravity varies over the surface of the earth. At London the value of g is 32·1912; at the equator, 32·088; at Spitzbergen, 32·252. This is due to two causes. The centrifugal force at the equator is very great, and balances a certain portion of the force of gravity. At the poles centrifugal force is nothing, and therefore gravity acts with full force there. Again, places on the equator are farther from the centre of gravity of the earth than places at the poles; therefore, gravity acts upon them less powerfully. A point on the equator is 13 miles farther from the centre than the north pole, the equatorial diameter being 7,939, the polar, 7,913. Therefore, so far as this cause is concerned, the force of gravity at the equator is to its force at the pole as $\left(\frac{7913}{2}\right)^2 : \left(\frac{7939}{2}\right)^2$.

EXERCISES.

1. Explain what is meant by centrifugal force, and show how it is measured.

2. Show that when a body moves in a circle its centrifugal force is proportional to the square of the velocity divided by the radius.

3. A body, whose mass is 5 lbs., moves with a uniform velocity of 80 ft. per second, in a circle whose radius is 10 ft. What force must act upon it, and in what direction? In what units is your answer estimated? *Answer.* 100 lbs; 3,200 absolute units.

4. A body moves in a circle, radius 3 feet, with velocity of 10 feet per second. Find the centrifugal force. *Answer.* $1\frac{1}{24}$ of W; $33\frac{1}{3} \times$ mass.

5. A body weighing 20 lbs. moves in a circle of radius 20 feet with velocity of 20 feet per second. Find the centrifugal force. *Answer.* $12\frac{1}{2}$ lbs.; 400 absolute units.

6. If a body moves in a circle, whose radius is 25 yards, with a velocity of 380 feet per second, find the centrifugal force. *Answer.* $60\frac{1}{6}$ of W; $1{,}925 \times$ mass.

7. A weight of 7 tons moves with a velocity of 30 miles per hour on a circle whose radius is 500 yards. Find the centrifugal force in tons. *Answer.* $\cdot 282\frac{1}{3}$ tons.

8. A body 1 lb. weight performs 100 revolutions per second, in a circle of a foot radius; required its centrifugal force in pounds. *Answer.* 12337·06 lbs.

9. If a railway carriage, weighing 8 tons, moving at the rate of 40 miles per hour, describe a portion of a circle whose radius is 400 yards, calculate the centrifugal force in tons. *Answer.* ·717 tons.

10. Explain the action of a pendulum, and show that the time of vibration is proportional to the square root of the length.

11. Find the time of vibration of a pendulum of given length, and show under what circumstances it varies as the square root of the length.

12. What is a pendulum? If a pendulum vibrates seconds, and its length is increased by an inch, what will then be the time of its vibration? *Answer.* 1·0127 second.

13. A pendulum which vibrates seconds is lengthened one inch; how many seconds will it lose in a day? *Answer.* 1,083.

14. When a pendulum is set vibrating why does it not stop when it comes to the lowest point? A chandelier hangs from a high roof, and is itself 20 feet from the ground. It makes five vibrations in 41 seconds. What is the height of the roof? *Answer.* 239·31 feet.

15. If a small heavy ball is suspended by a fine thread 12 feet long, find how many small oscillations it will make in a minute. *Answer.* 31·2.

16. A pendulum which vibrates seconds at the earth's surface is taken down a mine which is one mile deep; will it gain or lose, and how many seconds, in a day? Take the earth's radius equal to 4,000 miles. *Answer.* It loses 10·8 seconds per day, nearly.

17. What is the *centre of oscillation*? Give an illustration of the proposition that the centres of oscillation and suspension are convertible, explaining it fully.

18. A small ball is suspended by a thread from a point, and makes small oscillations. The thread is shortened by 2 feet, and now the number of oscillations per minute is increased in the ratio of 4 : 3; find the original length of the thread. *Answer.* $4\frac{4}{7}$ feet.

CHAPTER XII.

WORK AND ENERGY.

213. To estimate the amount of labour involved in any operation the following method is adopted :—

The work involved in lifting 1lb. through a height of 1 foot is called a **foot-pound**, and is taken as the unit of work. It is evident that the work of lifting a weight W lbs. through 1 foot will be W $\times$ the work of lifting 1lb. through 1 foot, or W foot-pounds. If now we lift it through another foot, the work is another W foot-pounds, making altogether W $\times$ 2 foot-pounds: and, generally, if we lift it through h feet, we have done a work of W $\times$ h foot pounds.

Now, suppose we move the body through s feet in any direction (not necessarily the vertical), and that throughout the operation we are exerting upon it a uniform force of F lbs. in that direction, the work is manifestly the same to us as if we had lifted a weight of F lbs. through a vertical height of s feet.

$\therefore$ the work done = F $\times$ s. Thus we see that the work of horizontal or other forces can be estimated in foot-pounds, and that the work done by any force is found by multiplying the number of pounds in the force by the number of feet through which its point of application has been moved in the direction of the force. [Observe that we cannot multiply a pound by a foot, and obtain a foot-pound. We only multiply the *number* of units of force by the *number* of units of length, and the resulting *number* will evidently be equal to the number of units of work done by the force.]

The same method of measuring work is often employed with other units: thus the work of lifting a ton through a foot may be taken as the unit, and called a **foot-ton.** In the decimal system the work of lifting a gramme through one centimetre is employed, and is called a **gramme-centimetre.**

214. If we carry a pound weight to the top of a mountain, it exerts less weight than before, because the effect of gravity upon it is less; and thus the foot-pound obtained by lifting *it* through one foot varies from place to place. If, therefore, we take a foot-pound as our unit of work, we understand by a pound a *pressure* equal to that produced by the brass pound at the level of the sea; and the work of lifting the brass

pound through 1 foot at the top of a mountain is not a foot-pound, because the weight is no longer exerting a pressure of a pound.

To avoid this difficulty, absolute units may be employed, the *poundal* instead of the pound, and the work estimated in *foot-poundals*. And in the Metric system, we may take the work of lifting a *dyne* $\left(\frac{1}{g}\text{ of a gramme}\right)$ through a centimetre; this unit is called an *Erg*.

215. An agent is said to do work when it causes the point of application of the force it applies to move through a certain space in opposition to resistance. The efficiency of any agent is measured by the number of units of work done in a given time. If n = the nnmber of agents or workers, w = work done by each in one minute, t = number of minutes, Work = $w\,n\,t \therefore w\,n\,t = \text{F}\,s$.

If any *four* of these quantities be given, the fifth can be found.

216. **Horse-Power.**—In reckoning the work done by his steam-engines, Watt compared it with that done by a horse. He considered that a horse in good condition, working 8 hours daily, could raise 33,000lb. 1 foot high per minute. Hence he reckoned a "horse-power" as = 33,000 foot-pounds per minute.
= 550 „ per second.

217. **Energy** means the power of doing work; and any body having a power of doing work is said to possess energy. Thus a bullet shot from a gun, a stone thrown from the hand, a moving stream, a gale of wind, all have the power of overcoming resistance, and therefore possess energy.

If a stone weighing 1lb. be thrown upwards with a velocity of 32 feet per second it will rise to a height of 16 feet. Its energy at starting, therefore, is capable of doing 16 units of work. If it be thrown with a velocity of 64 feet it will rise 64 feet; if its velocity be 96 feet per second it will rise 144 feet. Thus by doubling the velocity the height to which the body rises is increased four times; by trebling the velocity the height is increased nine times. The power of doing work or the energy is therefore *proportional to the square of the velocity.*

Again, if a stone 2lbs. weight be thrown up with a velocity of 32 feet per second, it will riso 16 feet, the same height as the smaller body; but in raising it twice as much work is done. So also a stone 3lbs. weight will rise to the same height but three times as much work will be done in raising it. Hence

the power of doing work or energy is *proportional to the mass.*

In all these cases we omit the resistance of the atmosphere, or suppose the experiments to be made *in vacuo.*

218. If a force F acts upon a body of weight W in a certain direction in which W is already moving with velocity u, and if the resulting acceleration be f, velocity v, and space s, we have—

$$v^2 = u^2 + 2fs \quad \therefore \frac{v^2 - u^2}{2} = fs.$$

Multiply both sides by $\frac{W}{g}$; then

$$\frac{W(v^2 - u^2)}{2g} = \left(\frac{W}{g} \times f\right) s = Fs = \text{the work done}$$

by F in foot-pounds . . . (i)

Hence the work done by a force may be estimated by the alteration which it makes in the velocity of a body of known weight upon which it acts, as well as by the distance through which its point of application moves; and the work done in changing the velocity of a body of weight W from u to v is the same, whether it be accomplished by a small force or by a large one: for we can find it from the left-hand member of the equation above, without knowing what force F was employed.

If we suppose $u=0$, then $\frac{Wv^2}{2g}$ = work done in creating a velocity v in the body of weight W; and if we wish to stop this moving body we must perform upon it an amount of work $= -\frac{Wv^2}{2g}$; *i.e.*, $\frac{Wv^2}{2g}$ in the opposite direction; for then the initial velocity is v, and the final 0; and equation (i) gives—

Work done in stopping $W = \frac{W(0^2 - v^2)}{2g}, = -\frac{Wv^2}{2g}.$

Suppose that in stopping W, we used a constant force F through a distance s; then $-\frac{Wv^2}{2g} = Fs$; but all the time we press the body with force F, it presses us with a force equal and opposite; *i.e.*, with a force $-F$ through space s; $\therefore$ it does work upon us $= -F \times s$.

But $Fs = -\frac{Wv^2}{2g} \therefore -Fs = \frac{Wv^2}{2g}.$

In other words, if W is moving with velocity v, it will do a work$=\frac{W v^2}{2g}$ foot-pounds before it can be stopped; this expression is called the **energy or accumulated work** of the moving body.

Sometimes our information is such that we cannot determine the whole work done upon a body by means of the equation *Work* $=$ F s, or by means of the equation *Work* $= \frac{W(v^2-u^2)}{2g}$, but must consider two portions of the work separately, one by each equation.

Example.

A waggon weighing 3cwt. is drawn from rest through a distance of a mile, and is then observed to be moving at the rate of $7\frac{1}{2}$ miles per hour. Find the whole work that has been done upon it, the friction being$=$a force of 60lbs.

(i.) Using *Work*$=$F s, we get for the work done in overcoming friction

Work$=60\times5280=316{,}800$ foot-pounds.

(ii.) Using *Work*$=\frac{W(v^2-u^2)}{2g}$, we get for the work done in creating the velocity of $7\frac{1}{2}$ miles per hour, *i.e.*, 11 feet per second, *Work*$=\frac{336\times(121-0)}{64}=$ 635·25 foot-pounds $\therefore$ the total work done upon the body$=316{,}800+635{\cdot}25=317{,}435{\cdot}25$ foot-pounds.

*219. A body of weight W is drawn *up* a rough inclined plane by a constant force of P lbs. in such a manner that its velocity is increased from u to v feet per second while passing over s feet. It is required to find the work done by P in foot-pounds.

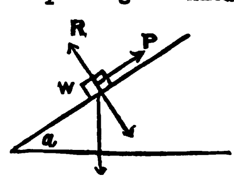


FIG. 83A.

The force W may be resolved into W sin. a down the plane, and W cos. a at right angles to the plane. Let μ be the co-efficient of friction between the body and the plane, and r the pressure between them. The only motion which takes place is along the plane.

There is no motion at right angles to it $\therefore$ the forces at right angles to the plane balance each other.

$\therefore$ R = W cos. a when in motion, as well as when at rest, and the friction acts down the plane, and is $= \mu$ R $= \mu$ W cos. a $\therefore$ resolving along the plane we have P upwards and W sin. $a + \mu$ W cos. a downwards. These do not balance, but P is the greater, and $\therefore$ an increase of upward velocity takes place. But, if F be the force which causes an increase of velocity from u to v in a body of weight W whilst passing over a space s, we have

$$Fs = \frac{W(v^2 - u^2)}{2g} \text{ by our formula.}$$

But the F of the formula is not our force P in the present case, because P is not *wholly* employed in increasing the velocity, but partly in overcoming the downward tendency of W along the incline. The F of the formula is the resultant of P and W sin. $a + \mu$ W cos. a, or P $-$ W(sin. $a + \mu$ cos. a).

$\therefore$ the formula gives

$$\{P - W(\sin. a + \mu \cos. a)\} \times s$$
$$= \frac{W(v^2 - u^2)}{2g} \therefore Ps = W(\sin. a + \mu \cos. a)\, s + \frac{W(v^2 - u^2)}{2g}$$

This therefore is the work done by P : for P is the force in pounds, and s the distance traversed under it in feet.

$\therefore$ Ps is P's work in foot-pounds.

*220. **Corollary.**—If no change of velocity takes place (*e.g.* a train going uniformly 30 miles an hour up an incline) then $v = u$, and

$$\frac{W(v^2 - u^2)}{2g} = 0$$

$$\therefore Ps = W(\sin. a + \mu \cos. a)\, s$$

In other words, if P exactly balances W (sin. $a + \mu$ cos.) whilst R balances W cos. a, the condition of the body is the same as if no forces acted on it, and $\therefore$ by the First Law of Motion it will remain at rest if it is at rest, and if in motion it will move in a straight line with uniform velocity.

*221. If a body be thrown vertically up with any given velocity u, the space it describes before coming to rest is = the space it would have taken to acquire that velocity if it had fallen from rest : for in the first case we get

$$0 = u^2 - 2\,g\,s \therefore s = \frac{u^2}{2\,g}.$$

And in the second case,

$$(-u)^2=0+2\,g\,s \therefore s=\frac{u^2}{2\,g}.$$

The same holds good when neither the initial nor the final velocity is zero: and when the force acting is not the weight, but any constant force F, so that the acceleration is f, not g (f being of the same constant nature as g, but not the same magnitude). For (a) let the velocity of W be *increased* from u to v by F: then

$$v^2=u^2+2\,f\,s \therefore s=\frac{v^2-u^2}{2\,f}=\frac{\mathrm{M}\,(v^2-u^2)}{2\,\mathrm{M}f}=\frac{\frac{\mathrm{W}}{g}(v^2-u^2)}{2\,\mathrm{F}}$$

$$=\frac{\mathrm{W}\,(v^2-u^2)}{2\,g\times\mathrm{F}} \quad . \quad . \quad . \quad \text{(i)}.$$

(b) Let the velocity of W be *decreased* from v to u by F, then $u^2=v^2-2\,f\,s$ (v is now initial, and f negative acting the opposite way to v)

$$\therefore s=\frac{v^2-u^2}{2\,f}=\frac{\mathrm{W}\,(v^2-u^2)}{2\,g\times\mathrm{F}}, \text{ as before.}$$

$\therefore$ space s is the same in both cases.

If instead of F a force F′ had produced the same alteration of velocity, and s' had been the space described, we should have had

$$s'=\frac{\mathrm{W}\,(v^2-u^2)}{2\,g\times\mathrm{F}'} \therefore \mathrm{F}'\,s'=\frac{\mathrm{W}\,(v^2-u^2)}{2\,g}=\mathrm{F}\,s.$$

But F F′ are forces in lbs. and s s' distances in feet: $\therefore$ F s, F′ s' represent foot-pounds. Hence we see that whatever force is employed to do the work, the raising the velocity of W from u to v (or *vice versâ*) requires a fixed number of units of work, viz., $\frac{\mathrm{W}\,(v^2-u^2)}{2\,g}$. A great force F will do this work in a short distance s, and a small force F′ will take a great distance s', but always such that $\mathrm{F}\times s=\mathrm{F}'\times s'$, and each $=\frac{\mathrm{W}\,(v^2-u^2)}{2\,g}$ foot-pounds.

222. The energy thus shown by a moving body is called **energy of motion,** or **kinetic energy.**

223. If a stone be thrown upwards with a velocity of 32·2 feet (9·8 metres) it will rise to a height of 16·1 feet and then stop; its energy is exhausted, and it begins to yield to

wheel must make one turn; therefore to raise the weight a distance equal to the circumference of the axle, the power must descend a distance equal to the circumference of the wheel; that is, P's journey : W's journey : : circumference of wheel : circumference of axle, or as W : P.

(3) *The Inclined Plane.*—If the power acts along the length of the plane (Fig. 49) in order to raise the weight through the vertical height of the plane, the power must ascend a distance equal to the length of the plane. Therefore $h : l$: : distance traversed by R : distance traversed by P. But $h : l$: : P : W. Therefore P : W : : W's journey : P's journey.

(4) *The Wedge* (Fig. 53).—While the power moves forward through the length of the wedge, R is moved through half the width of the back. Therefore R's journey : P's journey : : ½ back : length. But P : R : : ½ back : length. Therefore P : R : : R's journey : P's journey.

(5) *The Screw* (Fig. 54).—While the power travels round its circle, the resistance travels through the distance between two threads. Therefore P's journey : R's journey : : circumference of circle : pitch of screw. But P : R : : pitch : circumference. Therefore P : R : : R's journey : P's journey.

(6) *The Pulley.*—In Fig. 59, A, B, it will be seen that to raise R one inch, each of the cords which support it must be shortened one inch, and two inches of cord must run over the fixed pulley; hence P must move two inches. Therefore P : R : : R's journey : P's journey. In C, P must move three inches. It is needless to go through all the systems of pulleys; one example will be enough, but the student will do well to test the principle in the other cases also. In the first system, I., Fig 60, in order to raise W one inch, each of the cords which bear it must be shortened one inch; therefore the lowest pulley but one must rise three inches. That this may be so, each cord which supports it must be shortened three inches, or the third pulley must rise nine inches. For this purpose each part of the cord supporting it must be shortened nine inches, and 27 inches of cord must pass over the fixed pulley. P will therefore descend 27 inches, *i.e.*, P moves 27 times as far as W, or the journeys are inversely proportional to the forces.

228. **Modulus of a Machine.**—The useful effect or modulus of a machine is the fraction which expresses the value of the useful work done, compared with the total work

applied to the machine, which is denoted by unity. Thus, if a force of 100lbs. applied to a machine produces a useful effect, represented by 70lbs., the modulus is $\frac{7}{10}$ or ·7. The work lost in overcoming friction, heating, and wearing away the machine itself, &c., depends on the nature and extent of the rubbing surfaces, the mode of motion, &c. In some machines, as the screw and inclined plane, the work lost is very great, and the modulus consequently a fraction of small value.

EXERCISES.

1. What is a *horse-power?* How many horse-powers will it take to raise 4 cwt. per minute from a pit whose depth is 80 fathoms? *Answer.* 6·516.

2. A man works on a machine in such a manner as to do 1,000,000 units of useful work in a day of 8 hours. The machine is so arranged that he can lift a weight of 5 cwt. How long will it take him, working at his average daily rate, to lift that weight through a height of 100 feet? *Answer.* ·448 hours.

3. A bricklayer's labourer with his hod weighs 170lbs. He puts into the hod 20 bricks weighing 7lbs. each, and then walks up a ladder to a height of 30 feet. How many units of work does he do? And, if he can do 1,500,000 units of work per day, how many bricks will he take up the ladder in a day? *Answer.* 9,300—3,220 nearly.

4. How many gallons of water would a steam-engine of 10 horse-power raise from a depth of 200 fathoms in an hour? (A gallon of water weighs 10lbs.) *Answer.* 1,650.

5. If a man can do 900,000 units of work in a working day of 9 hours, at what fraction of a horse-power does he work on an average? *Answer.* $\frac{5}{99}$.

6. How many units of work must be expended in raising from the ground the materials for building a uniform column 66 feet 8 inches high, and 21 feet square, a cubic foot of brickwork weighing 1 cwt.? (The whole weight must be considered as raised the height of the centre of gravity of the column, *i.e.*, 33 feet 4 inches.) *Answer.* 109,760,000 units.

7. State the principle of work, and apply it to find the relation between the power and the weight in the third system of pulleys, the strings being parallel, and the number of pulleys n.

8. A body weighing 100lbs. moves without rotation at the rate of 20 miles an hour: find the number of units of work accumulated in it. If from the instant under consideration the body slides along a rough horizontal plane, find how far it will go before coming to rest, the *coefficient of friction* between the body and the plane being ·05. *Answer.* 1,334·4 foot-pounds; 266·8 feet.

9. A mass of 6 tons moves at the rate of 10 feet per second: find the number of units of work accumulated in it. If the mass is

acted on by a force of 20lbs. in a direction opposite to the direction of its motion, how far will it move before being brought to rest? *Answer*. 21,000 units; 1050 feet.

10. A body weighing 50lbs. is moved from a state of rest, and is found after a certain time to have a velocity of 10 feet per second: how many units of work must the force causing the motion have done over and above those expended on the resistance? *Answer*. 77·6 units.

11. A railway truck weighs 12 tons; it is drawn from rest by a horse through a distance of 50 feet, and is then moving at the rate of 3 miles an hour; if the resistances amount to 8lbs. per ton, how many units of work must the horse have done on the truck? *Answer*. 12,931 units.

12. The mass of a body is 10lbs. At a given instant it is moving at the rate of 40 feet per second; from this instant a constant force is made to act on it in a direction opposite to that of the motion, which brings it to rest after it has described 18 feet. What is the magnitude of that force? *Answer*. $444\frac{4}{9}$ absolute units, or $13\frac{8}{9}$lbs.

13. A body weighing 23lbs. has its velocity increased by 7 feet per second in any second of its motion: find the magnitude of the force producing this acceleration. How many pounds of matter would this force support against gravity in a place where $g = 32·2$? *Answer*. 161 poundals; 5lbs.

CHAPTER XIII.

IMPACT, OR COLLISION OF BODIES.

229. **An impulsive force,** as already stated, is one which produces a finite change of motion in an indefinitely short time. When a cricket-ball is struck by a bat its original velocity is destroyed, and a new velocity is imparted to it in a contrary direction: both velocities may be very great, and yet the whole process takes place in an extremely short time. The force acting on the ball differs from such a force as gravity —which produces a comparatively small velocity in a much longer time—not in kind but in degree. The change of motion seems to be produced instantaneously, but it really requires some time, though the period is too short for our observation. An impulsive force, therefore, is not measured in the same way as other forces producing motion—by the momentum generated in a given time—but by the whole momentum which it generates. Impulsive forces are those with which we are concerned in the impact or collision of bodies.

230. In treating of impact we shall consider the bodies as being smooth spheres of uniform density, and free from any motion of rotation. The line joining the centres of the spheres at the instant of impact is the *line of impact.* When the centres of both spheres move along this line before collision the impact is *direct,* otherwise it is *oblique.*

231. The theory of impact and of impulsive forces furnishes an illustration of the third law of motion, thus enunciated by Newton: If one body impinges on another, and changes the motion of the other body, its own motion experiences an equal change in the opposite direction. Motion is here to be understood as measured by *momentum.*

232. **Direct Impact.**—If one inelastic body, A, overtakes another, B, moving in the same line, a certain compression of the particles of each will take place, during which A will lose velocity, and B will gain. The *momentum* lost by A will be

equal to that gained by B. As soon as the velocities become equal the action will cease and the bodies will move on together with a common velocity.

Let u and u' be the velocities of A and B before impact, and V their common velocity after impact. Then

$$\text{Momentum lost by } A = Au - AV,$$
$$\text{Momentum gained by } B = BV - Bu^1,$$
$$\therefore Au - AV = BV - Bu^1,$$
$$\therefore AV + BV = Au + Bu^1,$$
$$\therefore V = \frac{Au + Bu^1}{A + B}.$$

That is, the velocity after impact is found by dividing the sum of the momenta before impact by the sum of the masses. Hence the total momentum is the same after impact as before.

233. Suppose the bodies to be moving in opposite directions along the line of impact so as to meet each other. Then one velocity must be considered as negative, suppose $-u^1$. Then

$$\text{Momentum lost by } A = Au - AV$$
$$\text{Momentum gained by } B = BV + Bu^1$$
$$\therefore Au - AV = BV + Bu^1$$
$$\therefore V = \frac{Au - Bu^1}{A + B}$$

Or, the velocity after impact is found by dividing the difference of the momenta before impact by the sum of the masses. If $Au = Bu^1$, the velocity after impact will be $= 0$, *i.e.*, the bodies will both be brought to rest.

234. If the bodies A and B are elastic, compression will go on as before until the common velocity is acquired. At that instant the bodies begin to recover their figure, and they exert pressure against each other as long as they are in contact. A loses momentum and B gains momentum during expansion as well as during compression. If the momentum lost by A and gained by B during expansion be equal to that gained and lost during compression, the bodies are *perfectly elastic;* if less, they are *imperfectly elastic.* If we call the momentum exchanged during compression M_1, and that exchanged during expansion or restitution M_2, then the fraction $\frac{M_1}{M_2}$ is called the *coefficient of elasticity* for these bodies, and is denoted by the letter e. The coefficient of elasticity is

therefore equal to $\frac{\textit{velocity of recoil}}{\textit{velocity of impact}}$. e is either equal to 1, when the elasticity is perfect, or to 0, when the bodies are inelastic, or to some intermediate value, when the elasticity is imperfect.

During compression the velocity of A becomes $\frac{Au + Bu^1}{A + B}$; its momentum therefore becomes $A. \frac{Au+Bu^1}{A+B}$, and its momentum before impact was Au. The momentum exchanged up to the end of compression is thus $= Au - A. \frac{Au + Bu^1}{A + B} =$
$\frac{A^2u + A\,Bu - A^2u - A\,Bu^1}{A+B} = \frac{AB\,(u-u^1)}{A+B}$.

The momentum exchanged during restitution $= e.\frac{AB(u-u^1)}{A+B}$; and the whole momentum exchanged $= (1 + e)\frac{AB(u-u^1)}{A+B}$. If v and v^1 be the velocities of A and B after impact, the momentum lost by A may also be expressed by Au-Av, and that gained by B by Bv^1-Bu^1.

$$\therefore Au - Av = \frac{(1 + e)\,AB\,(u - u^1)}{A + B},$$

$$\text{and } Bv^1 - Bu^1 = \frac{(1 + e)\,AB\,(u - u^1)}{A + B},$$

$$\therefore v = u - \frac{(1 + e)\,B\,(u - u^1)}{A + B} = \frac{Au + Bu^1 - Be\,(u - u^1)}{A + B} \quad (1).$$

$$\text{and } v^1 = u^1 + \frac{(1 + e)\,A\,(u - u^1)}{A + B} = \frac{Au + Bu^1 + Ae\,(u - u^1)}{A + B} \quad (2).$$

The value of e is the same for the same substances, and is independent of the velocity or impressing force. In steel balls $e = \frac{5}{9}$; in glass $\frac{15}{16}$.

If the bodies are *perfectly elastic* $e = 1$, and (1) becomes

$$v = \frac{Au + Bu^1 - Bu + Bu^1}{A + B} = \frac{(A - B)\,u + 2\,Bu^1}{A + B}.$$

$$\text{and (2) } v^1 = \frac{Au + Bu^1 + Au - Au^1}{A + B} = \frac{2\,Au - (A - B)\,u^1}{A + B}.$$

If the bodies are *non-elastic*, $e = 0$; and (1) becomes

$$v = v^1 = \frac{Au + Bu^1}{A + B}, \text{ as in article 232.}$$

235. **Oblique Impact.**—In oblique impact both bodies are moving before impact along lines inclined to the line of impact suppose at angles a and a^1. The velocities along these lines may be resolved into velocities along the line of impact and at right angles to it. These component velocities may be combined separately, and the resultant components along the line of impact and at right angles to it obtained, and these resultants may again be compounded so as to give the ultimate direction and motion of each body after impact; and thus the problem is reduced to one of direct impact with diminished velocities.

236. **Impact on a Plane Surface.**—If an elastic spherical body moving along the oblique line A C impinge on the smooth hard surface F G (Fig. 84) it will rebound along some oblique line. If both the body and the plane are perfectly elastic, the body will, after impact, move along the line C E, making the angle E C D equal to A C D. But if they are not both perfectly elastic, the body will move along some other line C B, so that tan B C G : tan A C F : : e : 1 or tan B C G = e tan A C F.

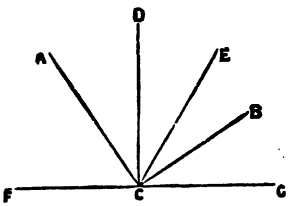


FIG. 84.

237. A body starting from a given point A, impinges against a given smooth plane F C (Fig. 85) and then passes through another given point B; determine the direction of incidence, e being known.

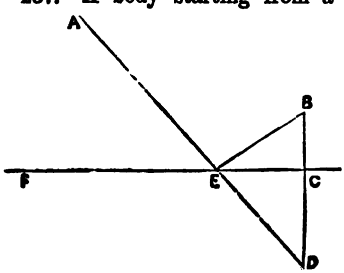


FIG. 85.

Draw B C perpendicular to the plane, and produce it, making C D $= \frac{1}{e}$ B C. Join A D, cutting the plane in E, and join E B. A E is the direction of incidence, and E B the direction of reflection.

238. **Centre of Percussion.**—In Fig. 83, if A B be a body suspended so as to turn freely round an axis passing through S, and be struck horizontally at O, it will vibrate in the same

time as if it were a simple pendulum of length O S, and with its whole mass concentrated at O. In this case, the whole energy of the blow will be expended in producing rotation. But if it be struck above or below O, part of the blow will be expended in producing a jar or strain on the axis, and the remainder in producing rotation. O is therefore the most advantageous point at which to strike, if we wish to make the body rotate; it is called the *centre of percussion* with reference to the axis passing through S, and we see that it is the same as the centre of oscillation.

If A B be employed as a striking body and be held at S, then O is the best point to strike at, so as to produce the greatest effect on the body struck. If, for instance, A B represents a bat used for striking a cricket ball, the ball should be struck by O; if it be struck with any other part of the bat, the hands and arms will be painfully jarred.

239. **The Ballistic Pendulum** is a machine employed to ascertain the velocity with which a shot leaves the mouth of a cannon. It consists of a large heavy block of wood suspended so as to move freely round a horizontal axis. The shot is fired into the block at rest, and remains in it. The block is set in motion, and vibrates round its axis through a certain angle, which is measured by means of a graduated arc placed under the pendulum. The velocity of the pendulum is calculated from this arc, and the velocity of the ball is obtained from that of the pendulum. It is now more usual to suspend the gun itself, and to calculate the velocity from the amount of the recoil.

EXERCISES.

1. A ball falls from a height of 100 feet, and rebounds 70 feet; what is the coefficient of elasticity? *Answer.* ·836 ($=\sqrt{.7}$).

2. If a non-elastic body weighing 70 lbs. moving to the southward with a velocity of 70 feet per second come in contact with another non-elastic body which weighs 80 lbs. and is moving southward with a velocity of 50 feet per second, so that the two bodies coalesce and move on together, in what direction will they move and with what velocity? *Answer.* 59⅓ feet per second southward.

3. Suppose the bodies in the last question to be moving in opposite directions, find the velocity after impact. *Answer.* Six feet southward.

4. An inelastic body moving with velocity v impinges on another inelastic body of the same mass at rest; what is the velocity after impact? *Answer.* $\frac{1}{2}v$.

5. A body weighing 10 lbs. and moving 12 feet per second impinges on a body weighing 6 lbs. with velocity of 5 feet per

second; find the velocities after impact, if $e = \frac{1}{2}$, both bodies moving in the same direction. *Answer.* $8\frac{1}{16}$ feet and $11\frac{9}{16}$ feet per second.

6. A body of mass 15 lbs., with velocity of 12 feet per second, impinges on a body of mass 20 lbs. with a velocity of 6 feet per second in the same direction; find the common velocity at end of compression, and their joint energies just before impact and at end of compression. *Answer.* $8\frac{4}{7}$ feet; 1440 foot-poundals; $1285\frac{5}{7}$ foot-poundals.

7. Two bodies of 10 lbs. and 12 lbs. move in the same direction with velocities of 8 feet and 6 feet per second; find their velocities after impact (1) if the bodies are inelastic; (2) if they are perfectly elastic; (3) if the coefficient of elasticity be $\frac{1}{4}$. *Answer.* (1) $6\frac{10}{11}$ feet; (2) $7\frac{9}{11}$ and $5\frac{9}{11}$; (3) $7\frac{3}{22}$ and $6\frac{7}{11}$

8. Two non-elastic bodies weighing 10 lbs. and 8 lbs. move along the same line in opposite directions with velocities of 8 feet and 10 feet per second; find their velocity afler impact. *Answer.* 0.

9. Two perfectly elastic bodies of equal mass move in opposite directions along the same line with velocities of 60 feet and 40 feet per second; find the velocities after impact. *Answer.* -40 and $+60$.

10. Masses of 10 and 12 kilogrammes move in the same direction with velocities of 20 and 16 metres per second; find the velocities after impact. *Answer.* $15\frac{7}{11}$ metres and $19\frac{7}{11}$ metres per second, if perfectly elastic.

HYDROMECHANICS.

CHAPTER XIV.

FLUID PRESSURE.

240. **Fluids** are substances whose particles are displaced by a very slight force, and are easily moved among themselves with so little friction that it may be practically disregarded, except in viscous liquids, like treacle. Hence they readily change their form, and adapt themselves to any vessel in which they may be placed. There is very little cohesion among the particles, and hence each particle of a fluid must be directly supported. In this respect they differ from ordinary solids. The slight friction distinguishes them from powders. If a quantity of sand be poured out on a horizontal table, it soon comes to rest and forms a heap, the motion of the particles being stopped by friction; but when water is poured out in the same way it flows out and spreads in a very thin film on the surface. Fluids comprise the two classes of liquids and gases. **Liquids** are capable only to a very small degree of being compressed or expanded; hence they are often called *non-elastic* fluids, though the name is not strictly accurate. Water is a common example. **Gases** are very compressible and very expansible, and are called *elastic* fluids, as air.

241. The facts and laws relating to the mechanical effects of liquids form the subject of **Hydromechanics**, which embraces the two branches—*Hydrostatics* and *Hydro-kinetics*. **Hydrostatics** treats of the conditions of equilibrium of liquids, and of the pressures they exert, both within their own mass and on the surfaces which limit them. **Hydro-kinetics** or **Hydraulics** treats of the motion of liquids and the effects resulting from this motion.

242. Liquids were formerly believed to be perfectly incompressible. This was supposed to be proved by the experiment of the Florentine Academicians: A globe of gold was filled with water, and subjected to enormous pressure. But it was found that no perceptible change could be produced in the form, and therefore in the content of the globe, until the water issued through the pores of the gold. They therefore

pressures on every point of the surface always passes. If the pressure at every point were equal like the force of gravity, the centre of pressure would coincide with the centre of gravity. But since the pressure increases with the depth, the centre of pressure is always, unless the surface be horizontal, lower than the centre of gravity.

262. The centre of pressure is evidently that point at which a single force might be applied which would balance all the pressures against the surface. Hence it may sometimes be found experimentally by ascertaining the point at which a prop will support the surface under pressure. Its position has to be carefully attended to in placing the hinges and bars of the lock gates of canals.

*263. In a few cases the centre of pressure is easily determined. Let A B (Fig. 99) be the section of a vertical rectangle pressed on one side by water. Since the pressure at every point is proportional to the depth, we may represent the pressures by lines equal to the depths and at right angles to A B. These lines may be taken as forming the triangle A B C, whose area therefore represents the total pressure acting on A B. This pressure will act through the centre of gravity of A B C, that is, through G, $\frac{1}{3}$ of the way up the line A D, and along the line G P. A B is only one line of section: every vertical line of the rectangle has its resultant pressure at the same depth $\therefore$ the resultant pressure on the entire rectangle acts at that depth and evidently, by symmetry, in the central section, or section through the c. g. of the rectangle. Hence P, the centre of pressure, is $\frac{1}{3}$ of the way up B A, which represents the middle line of the rectangle. The same holds good when the rectangle is not vertical.

*264. If the surface immersed be a triangle A B C (Fig. 100), with its base in the surface of the liquid, and its vertex downwards, the pressure on A B C will be represented in magnitude by the weight of the fluid contained in the triangular pyramid A B C D, formed by lines at right angles to A B C, and equal at each point to the depth of that point below the surface; and in its line of action by a straight line drawn from the c. g. of that pyramid at right angles to the plane of the triangle. Its centre of gravity, G, is found by joining b the centre of gravity of A B C with D, and making b G = $\frac{1}{4}$ b D (Art. 91). Hence the pressure acts through G at right angles on A B C at P, which is the centre of pressure. $bP=\frac{1}{4}bC$; but $bC=\frac{2}{3}Ca$. Therefore

P C$=\frac{3}{4}$ of $\frac{2}{3}$ of C$a=\frac{1}{2}$ Ca. That is, the centre of pressure is in the line joining the vertex C with a, the point of bisection of the base A B, and half-way up this line.

*265. If the triangle have its vertex in the surface of the liquid, and its base horizontal (Fig 100A), the pressure is represented by the weight of the liquid contained in the rectangular pyramid A B C E D, acting through its centre of gravity at right angles to the plane of the triangle. The centre of gravity is found by joining the *c.g.* of the base D E C B, viz., G, with A, making G$a=\frac{1}{4}$ G A, and hence K P$=\frac{1}{4}$ K A.

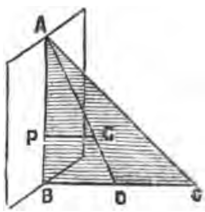


FIG. 99.

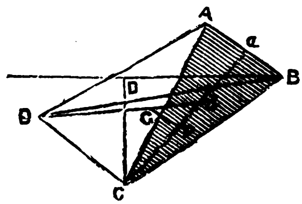


FIG. 100.

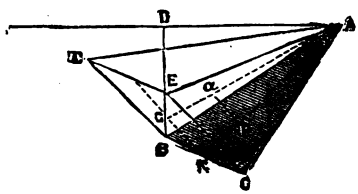


FIG. 100A.

Therefore, the centre of pressure is P, $\frac{1}{4}$ of the way up the medial line of the triangle.

EXERCISES.

1. What is the fundamental law of fluid pressure? How would you show it to be true? Describe an experiment.

2. Show that the surface of a fluid at rest must be horizontal.

3. When a vessel is filled with fluid, what is the measure of the pressure on its base (horizontal)? Give reasons for your answer.

4. In a fluid at rest under the action of gravity, show that the pressure varies as the depth.

5. What will be the pressure on a horizontal square inch in a vessel of water, at a distance of 25 inches below the surface? *Answer.* 6312·5 grains.

came to the conclusion that no compression had taken place: but they were wrong: for water is now known to be slightly compressible, and also slightly expansible. Thus, if a quantity of water be placed in a glass vessel, and the air exhausted, the water will expand about ·00005 in volume. And when a bottle, filled with fresh water and tightly corked, is let down to a considerable depth in the sea, and drawn up again, the water is found to be brackish, proving that the great pressure has driven in the cork and allowed salt water to enter.

243. **Pascal's Law of Transmission of Pressure through a Liquid.**—*Any pressure exerted on the surface of a liquid is transmitted equally in all directions, and acts at right angles to all surfaces in contact with it.*

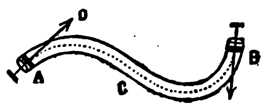


FIG. 86.

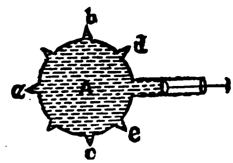


FIG. 87.

This law, which is the foundation of hydrostatics, can be shown to be true by various experiments.

(1.) If the bent tube A B (Fig. 86) be filled with water, and pressure be applied at A, an equal pressure must be applied at B to balance it. The pressure is therefore transmitted undiminished along the bent line A C B. If the tube were filled with a solid, like lead, the pressure at A would only be exerted in the direction A O.

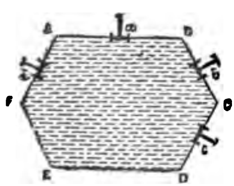


FIG. 88.

(2.) If the globe A (Fig. 87) be filled with water, and the piston* pressed in, water issues not only from the spout *a* in the direction of the pressure but from *b* and *c* at right angles to it, and from *d* and *e*, almost in an opposite direction.

(3.) Let the vessel A B C D E F (Fig. 88), closed on all sides, be filled with water. Let openings of equal size, *a*, *b*, *c*, *d*, have tubes closed by pistons inserted into them. Now, if a weight of 1 lb. be placed on *a*, an equal pressure

* A *piston* is a short cylinder or plug of wood or metal fitting closely to a hollow cylinder, and capable of being moved back and forward by a handle.

must be exerted against b and c to prevent them from being forced out. If the opening d be five times as large as a, a pressure of 5lbs. will be necessary to keep the piston at d in its place. a, b, c, d must be at equal distances below the surface.

244. On this principle, as applied to gases, depends the use of the **safety-valve** in the steam-engine. The valve is loaded with a weight somewhat less than the pressure which the boiler is constructed to withstand. As soon as the pressure of the steam becomes greater the valve is forced open, steam escapes, and the pressure falls.

245. **Hydrostatic Paradox.**—*Any pressure, however small, may, by transmission through a liquid, be made to support any weight, however great.* Thus, if the section of the pipe A (Fig. 89) be one square inch, and that of the vessel B, 40 square inches, a pressure of 1 lb. on A will support a pressure of 40lbs. on B. If the pressure on A be made more than 1 lb. the cover B will rise (but only $\frac{1}{40}$ of the distance through which A is forced downwards). We have an extension of this principle in Bramah's press. (Art. 377.)

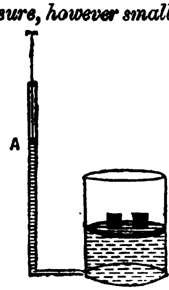


FIG 89.

246. **Unit of Area.**—In estimating pressures, we may take as unit of area the square inch, square foot, square yard, &c. We usually take the square inch. The pressure on any given area is the pressure on unit area $\times$ number of units in given area, *i.e.*, if the pressure be uniform. If the pressure be variable, the pressure at any point is measured by the pressure there would be upon a unit of area if the whole of that area were subject to the same pressure as exists at that point.

247. *If a fluid be at rest, any portion of it may be treated as a solid, without affecting the conditions of equilibrium.* This principle, which is self-evident, enables us to apply the laws of statics to fluids in equilibrium.

248. *The pressure at any point in a liquid at rest is the same in all directions.* The pressure at a point, in any direction, means the pressure on a unit of area which contains the point, and which is perpendicular to that direction. Let o be a point in the liquid contained in the vessel A B C D E F (Fig. 90), and let $a' b' c' d'$ be two discs, each containing a

unit of area, and parallel to B C, A F respectively. Let all the liquid be considered solid except the cylinder at $a\ b\ a'\ b'$.

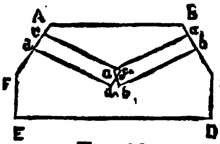


FIG. 90.

Then the pressure on $a\ b$ is the same as that on $a'\ b'$ by Article 243 (3). Again, consider all the liquid to become solid except the cylinder $c\ d\ d'\ c'$. The pressure on $c\ d$ is the same as the pressure on $c'\ d'$. Now the pressure on $a\ b$ is evidently the same as that on $c\ d$. Therefore the pressure on the disc $a'\ b'$ is equal to the pressure on $c'\ d'$. Therefore, at the point o the pressures are equal in these two directions; and similarly the pressures may be shown to be equal in every other direction. And since the distance of the point o from the sides of the vessel is not taken into account, the proposition is true *for any point.*

249. *The pressure of a liquid at rest is the same at all points in the same horizontal plane.* Let A and B (Fig. 91) be two points in the same horizontal plane in a liquid. Suppose the horizontal cylinder having those points in the centres of its ends to become solid.

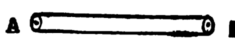


FIG. 91.

The pressure on the round surfaces and the weight of the cylinder have no tendency to produce motion in the direction of the axis. The only forces acting in that direction are the fluid pressures on the ends. Suppose the circular section indefinitely small, then the pressures at A and B may be considered as uniform. The pressure at A acts in the direction of A B, the pressure at B in the direction B A. But the cylinder is in equilibrium, therefore the pressure at A is equal to the pressure at B. That is, the pressures *on a unit of area* pressed all over as at the points are equal.

250. *The surface of a liquid at rest is a horizontal plane.* For, suppose the surface to be raised at O, and depressed at E (Fig. 92), then the particle O is urged downwards by its own weight, which may be represented by O G. This may be resolved into O Q perpendicular to the surface, and therefore balanced by the reaction of the liquid, and O R not opposed by any force, since friction does not exist. The particle will, therefore, move in the direction O R till the component O R disappears; that is, till gravity, E G, acts at right angles to

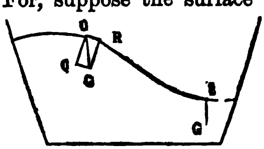


FIG. 92.

the surface; or, in other words, till the surface be horizontal. Every other particle will be acted on similarly. Hence the liquid will rest only when the surface is perpendicular to the direction of gravity, which position is called *horizontal*. If other forces besides gravity act on the particles of a liquid, the surface will be perpendicular to the resultant of all the forces acting. So, also, if two liquids which do not mix, are placed together in a vessel, their common surface will be horizontal.

251. Hence, water brought in pipes will rise to a height equal to that of its source. A knowledge of this principle enables us to dispense with the use of costly aqueducts, such as those constructed by the Romans, and to supply towns with water by means of underground pipes from some elevated position. It sometimes happens in nature that a layer of water collects between two strata of rocks at a considerable depth. At the lowest part it exists at great pressure; and if a well is dug down to reach it, the water will rise along it and flow over or rise above the ground in the form of a fountain. Such wells are called Artesian wells, from Artois, in France, where they were first dug in recent times.

252. **Levels** are instruments used to ascertain whether lines and surfaces are horizontal. The *water-level* consists of a horizontal tube of metal bent upward at its extremities, and terminating in two narrow glass tubes. Water slightly coloured is poured into the tube, and of course rises to the same height in both glass tubes, so that a view taken along both surfaces gives a horizontal line.

The *spirit-level* consists of a glass tube slightly curved upwards in the middle, and filled with alcohol or spirits of wine, except a small space occupied by a bubble of air. The tube is mounted on a stand whose lower surface is quite smooth. When this stand is placed on any horizontal surface the bubble will appear in the highest part of the tube, that is, exactly in the centre. This is a much more convenient instrument than the water-level.

253. It must be observed, however, that what is called a horizontal or plane surface is not really a plane surface, but the surface of a sphere. A B C (Fig. 93), part of a circumference of the earth, is the *real level* of a liquid surface. E B D, a tangent to that surface, is the

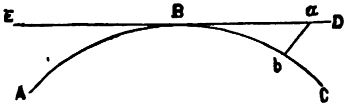


FIG. 93.

apparent level. The difference, $a\,b$, between the real and apparent levels is equal to about eight inches in every mile, and increases with the square of the distance. Thus, the difference in two miles is $2^2 \times 8 = 32$ inches; in three miles, $3^2 \times 8 = 72$ inches; in four miles, $4^2 \times 8 = 128$ inches. This difference has always to be attended to in making railways and canals.

254. The foregoing statement enables us to find—

(1.) From what height a given distance is visible at sea, or on a plain. $B\,b^2 \times 8 = a\,b$ in inches, or $B\,b^2 \times \frac{2}{3} = a\,b$ in feet; or $a\,b = \frac{2}{3}\,B\,b^2$. Therefore $\frac{2}{3}$ of the square of the distance in miles gives the height in feet.

(2.) The extent of the horizon, or the distance visible from any height. For $B\,b^2 \times 8 = a\,b \therefore B\,b^2 = \frac{a\,b}{8}$ inches $= \frac{3\,a\,b}{2}$ feet.

Therefore $B\,b = \sqrt{\frac{3\,a\,b}{2}} = \sqrt{a\,b + \frac{a\,b}{2}}$.

That is, to find how many miles are visible from any given height,—to the height in feet add its half, and extract the square root.

255. *The pressure at any point in a liquid at rest is proportional to the depth of that point below the surface of the liquid.*—Let A be a point in a liquid at rest (Fig. 94). Suppose the cylinder A B to become rigid. It is maintained at rest by two vertical forces, its own weight acting downwards, and the pressure at A acting upwards. Hence these forces are equal; that is, the pressure at A is equal to the weight of the cylinder A B. The pressure at C is equal to the weight of the cylinder C D. Hence the pressure is proportional to the depth. In this calculation and elsewhere unless otherwise stated, the atmospheric pressure is omitted.

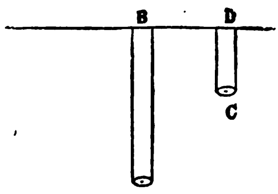


Fig. 94.

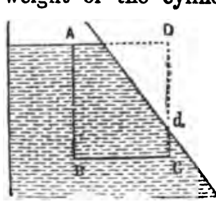


Fig. 95.

256. The pressure at C (Fig. 95) depends, not on its depth, Cd, below the side of the vessel; but on its depth C D, or its equal A B, below the free surface. For the pressure at C is equal

to that at B (Art. 249); and the pressure at B depends on the depth A B.

257. From (255) it appears that the *pressure of a liquid on a horizontal plane* is the weight of a column of liquid whose base is the given plane, and height the depth of the plane below the surface of the liquid. Hence the pressure on the bottom of a vessel filled with liquid is independent of the shape of the vessel, depending only on the area of the bottom and the depth of the liquid. Thus, in the vessels A, B, C (Fig. 96), with equal bases, and filled to the same depth, the bases sustain equal pressures, though the quantity of liquid varies greatly.

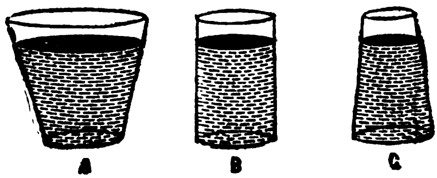


FIG. 96.

258. Let A B be any plane area obliquely immersed: draw vertical lines A A′, B B′, &c., forming a cylinder. The weight of the volume of liquid so enclosed is = the vertical component of the pressure on A B.

To find the whole pressure on A B: draw Aa at right angles to the plane and = A A′ · Bb = B B′, &c., and so on all round the boundary: then the weight of the volume of water contained in this new cylinder will be = to the whole pressure on A B. Find the c. g. of this water G, and draw G C at right angles to the plane: C is the centre of pressure.

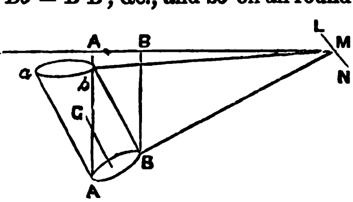


FIG. 97.

The points a, b, &c., can be shown to be in one plane, which will intersect the surface in the same straight line L M N as the plane of A B does.

If W be the weight of the liquid in a A Bb, then W cos. θ = weight in A′ B′ B A, where θ is angle B M B′, or inclination of the plane of A B to the horizon.

below the metacentre, and when the vessel is displaced gives the forces forming the couple a longer arm to work on.

277. Bodies consisting of material heavier than water may yet be made to float in water; provided they be made of such a shape that, without sinking below the surface, they displace a quantity of water equal to their own weight. Thus ships are made of iron; and while the water is excluded from the inside they float; but if the interior becomes filled with water, they sink. If a small needle is placed carefully on the surface of water it will float. This is due to the fact that the needle is not wetted by the liquid, and therefore, as explained in next chapter, a depression is formed round it. It thus displaces a quantity of liquid greater than its own bulk. On the same principle, some insects walk on water.

EXERCISES.

1. What is meant by *specific gravity?* A body which weighs 15 oz. in vacuo weighs 12 oz. in water; what is its specific gravity? *Answer.* 5.

2. If a cylinder floats vertically in water half immersed, what is its specific gravity? If now some fluid be mixed with water, so that the specific gravity of the mixture is 1·25, how much of the cylinder will be immersed? *Answer.* ·5; $\frac{2}{5}$.

3. Hiero gave a goldsmith a certain weight of gold to make into a crown. When the crown was brought back, it weighed exactly the same as the gold which had been given. But it was suspected that part of the gold had been replaced by an inferior metal. Archimedes discovered this for the king: give one method by which this could be done. [Let W_1 and W_2 be the weights of gold and alloy in the crown, W the weight of pure gold originally given, and also the total weight of crown; S_1 and S_2 the sp. grs. of gold and alloy referred to water, W^1 the weight of water displaced by the crown.

$$\text{Then } W_1 + W_2 = W \quad . \quad . \quad . \quad \text{(i.)}$$

$$\text{and } \frac{W_1}{S_1} + \frac{W_2}{S_2} = \frac{W^1}{1} \quad . \quad . \quad . \quad \text{(ii.)}$$

Substituting for W_1 from (i) in (ii) we get

$$\frac{W - W_2}{S_1} + \frac{W_2}{S_2} = \frac{W^1}{1}$$

$$\therefore S_2 (W - W_2) + S_1 W_2 = S_1 S_2 W^1$$

$$\therefore (S_1 - S_2) W_2 = S_1 S_2 W^1 - S_2 W$$

$$\therefore W_2 = \frac{S_1 S_2 W^1 - S_2 W}{S_1 - S_2} \quad . \quad . \quad . \quad \text{(iii)}$$

Now Archimedes knew W, W^1, and S_1; but did not know, except by guess, what material was used as alloy; so that he did not know

the value of S_2, and the equation (iii) remained with two unknowns and $\therefore$ indeterminate. It was only by guessing the substance used that he could find S_2, and so get W_2, the weight of base metal in the crown, and $W - W_2$, the weight of gold in it. For illustration, if Archimedes had reason to suspect that the alloy was silver, and if the weight of the crown was 1lb., and of the water it displaced ·062 of a lb., then from the table,

$$S_1 = 19{\cdot}35, \text{ and } S_2 = 10{\cdot}47.$$

$$\therefore W_2 = \frac{19{\cdot}35 \times 10{\cdot}47 \times {\cdot}062 - 10{\cdot}47 \times 1}{19{\cdot}35 - 10{\cdot}47}$$

$$= \frac{12{\cdot}561 - 10{\cdot}47}{8{\cdot}88} = \frac{2{\cdot}091}{8{\cdot}88} = {\cdot}235\text{lb.}$$

leaving ·765lb. for the gold of the crown.]

4. A vessel is full of water, a piece of wood weighing 3 lbs. is placed in it and floats; how many cubic inches of water will run over? *Answer.* 82·944.

5. A cubical box, whose side is 10 inches, floats in water; its weight is 10 oz. Neglecting the thickness of the material, how much mercury (specific gravity = 13·6) can be poured in before the water overflows the edges of the box? *Answer.* 72·255 cubic inches.

*6. A cone of wood whose axis is 8 inches and radius of base 2 inches, floats in water with its axis vertical. If its specific gravity be ·8, how much of the axis is immersed? *Answer.* 7·42654 inches.

7. A B C is a tube, bent so that the legs are parallel. Being held vertically upwards, 8 cubic inches of mercury are poured into C, and 12 cubic inches of water into A (the section of the tube being 1 sq. in.). Find the distances between the upper surfaces of the mercury and water and their common surface. The horizontal part is 2 inches long. *Answer.* Mercury, ·882 inch; water, 12 inches.

8. Find the weight of a stone which, when placed on a block of wood weighing 523 ounces, will just cause it to sink in water (specific gravity of the wood = ·596). For equilibrium, wt. of stone + wt. of wood = wt. of water displaced. Let W be wt. of stone in ounces, and V the volume of the wood in cubic feet: then

$$W + V \times {\cdot}596 \times 1000 = V \times 1 \times 1000 \text{ oz.}$$

But $V \times {\cdot}596 \times 1000 \text{ oz.} = 523$ oz. by the question.

$$\therefore V \times 1 \times 1000 = \frac{523}{{\cdot}596}$$

$$\therefore W + 523 = \frac{523}{{\cdot}596} = 877{\cdot}51.$$

$$\therefore W = 877{\cdot}51 - 523 = 354{\cdot}51 \text{ ounces.} \quad \textit{Ans.}$$

9. A cube whose edge is 12 inches floats in water, with its sides vertical and horizontal. One-fourth part is cut away by a plane parallel to the horizon, and then it is found to float with the part extant equal to twice the part which was before immersed. Find the weight of the cube. *Answer.* $17\frac{1}{32}$ lb.

10. Three fluids, whose specific gravities are 2, 3, 4, and their volumes as 3, 4, 5, are mixed; find the specific gravity of the compound. *Answer*. $3\frac{1}{6}$.

11. What are the conditions of equilibrium of a floating body? Show that they are necessary and sufficient.

12. How is that ships can be made of material heavier than water? Why do ships carry heavy ballast in the hold?

13. The specific gravity of cork being ·26, if 1oz. of cork is tied by a thread to the bottom of a vessel of water so as to be kept wholly under water, what is the tension of the thread? *Ans*. $2\frac{11}{13}$oz.

14. What is the *metacentre* of a floating body? Show how it is found, and explain its use in determining the stability.

15. When a body is immersed in a fluid, the weight lost is to the whole weight of the body as the specific gravity of the fluid is to the specific gravity of the body. Prove this.

CHAPTER XVI

SPECIFIC GRAVITY.—*Continued.*

278. In this chapter we shall treat of the various instruments in most common use for determining the specific gravities of solid and liquid bodies.

The Specific Gravity Bottle is employed for both liquids and solids. It is a glass bottle, fitted with a ground stopper, and capable of holding exactly 1,000 grs. of distilled water at the standard temperature. The stopper is perforated, so as to allow any liquid beyond that quantity to flow through.

279. *For a Liquid.*—The empty bottle is placed at one arm of a balance, and counterpoised by weights in the opposite scale. It is then filled with the liquid at the standard temperature, and balanced by weights added to the scale. The weight, W, so added, is the weight of the liquid in the bottle. But the same volume of water weighs 1,000 grains. The specific gravity of the liquid is therefore $= \frac{\text{weight of liquid}}{\text{weight of water}} = \frac{W \text{ grains}}{1000}$.

280. *For a Solid.*—Place a known weight, say 400 grains of the solid in the bottle, counterpoised as before. Fill the bottle with water, and weigh. Suppose the weight now to be 1,300 grains. There are therefore 900 grains of water in the bottle; but it only holds 1,000 grains of water when full; therefore the solid fills the place of 100 grains of water. Consequently, its specific gravity must be $\frac{400}{100} = 4$. Or, generally, if W = weight of water and the given solid filling the bottle, then W is = 1,000 grs. − weight of water displaced (W_1) + weight of body (W_2).

$$W = 1000 + W_2 - W_1 \therefore W_1 = 1000 + W_2 - W.$$

Therefore specific gravity $= \frac{W_2}{W_1} = \frac{W_2}{1000 + W_2 - W.}$

281. The paragraphs 269-271 contain the **principle of Archimedes**,* which may be thus enunciated: "*If a body be immersed (either wholly or partly) in a liquid, it is pressed upwards with a force equal to the weight of the liquid displaced,*

* Discovered by Archimedes in solving the problem contained in Exercise 3 at end of last chapter.

and whose point of application is in the same vertical line with the centre of gravity of the displaced liquid." If the body be of uniform density (homogeneous) its centre of gravity will coincide with the centre of gravity of the displaced liquid : if not it will be in the same vertical line.

282. Hence, a body which sinks in water displaces a quantity of water equal to its bulk and less than its weight. A body which floats displaces a quantity equal to its weight, but less than its bulk. The former loses a portion of its weight equal to the weight of the liquid displaced. The latter exerts no force on a string by which it is suspended.

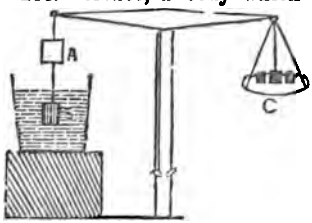


FIG. 103.

283. Let A (Fig. 103) be a hollow cylinder of copper, and B a solid cylinder which would just fill it. Let them be balanced by weights in scale C, and immerse B in water. The scale C will descend, on account of the upward pressure on B. Now fill A with water, and equilibrium will be restored, showing that the upward pressure on B is equal to the weight of the water in A, *i.e.*, of a mass of water equal in bulk to B, the body immersed.

284. **The Hydrostatic Balance** (Fig. 104) differs from the ordinary balance in having a counterpoise, C, attached to one end of the beam, from which the body B, whose specific gravity is to be ascertained can be hung. First weigh the body B in air:* suppose it to weigh 10oz. Then allow it to dip into the vessel of water ; the scale D will descend. Remove weights from D until the beam is again horizontal. The weight so removed, say 2oz., is equal to the weight of the water displaced. Therefore the specific gravity is

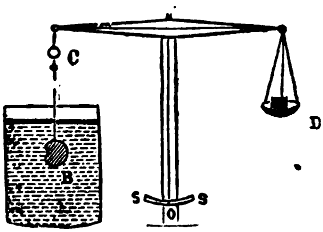


FIG. 104.

* Strictly speaking, the body should be weighed in vacuo : the weight in air is less than the true weight of the body by the weight of the air displaced. Except in very delicate investigations, however, the usual method is sufficiently accurate.

$\frac{10}{2}=5$. If W be the weight of any body in air, and w its weight in water—

$$\text{Specific gravity}=\frac{\text{weight in air}}{\text{loss in water}}=\frac{W}{W-w}$$

Example.

A piece of lead weighs 39·76 grammes in air, and 36·26 grammes in water; find the specific gravity.

$$\text{Specific gravity}=\frac{39{\cdot}76}{39{\cdot}76-36{\cdot}26}=39{\cdot}76\div3{\cdot}5=11{\cdot}36. \quad \textit{Ans.}$$

The horizontality of the beam is shown by the index pointing to O, on the scale S S.

If the body be lighter than water, attach to it some body heavy enough to sink it in water. Weigh the two together in air and water; the loss in weight, L, is equal to the weight of the water displaced by both. Then weigh the heavy body both in air and water; the loss in weight, l, is equal to the weight of the water displaced by the sinker. The difference of the two losses will be the weight of the water displaced by the given body. Thus, the

$$\text{Specific gravity}=\frac{\text{weight of body}}{\text{difference of two losses}}=\frac{W}{L-l}$$

Example.

A piece of cork weighing 12oz. is attached to a sinker of lead weighing 60oz.; their weight in water when joined is 16·72oz.; the lead alone weighs in water 54·72oz.: find the specific gravity of the cork.

$$\text{Specific gravity of cork}=\frac{12}{50}=\cdot24.$$

285. To determine, by the balance, the specific gravity of a liquid.—Take any heavy body which the liquid cannot dissolve or corrode. Ascertain its precise weight in air. Then weigh it, first in the given liquid, then in water, and observe its loss of weight in each case. Divide the loss in the liquid by the loss in water; the result is the specific gravity of the given liquid. The reason is evident from Example 1.

Examples.

(1) Thus, if a body weighing 500 grains in air weigh only 350 grains in the liquid, and 400 grains in water, the

specific gravity of the liquid is $\frac{150}{100} = 1 \cdot 5$. For a quantity of the liquid equal in bulk to the body weighs 150 grains; an equal quantity of water weighs 100 grains. Hence the

$$\text{Specific gravity} = \frac{\text{weight of liquid displaced}}{\text{weight of water displaced}} = 1 \cdot 5$$

(2) A body weighs 2,300 grains in air, 1,000 grains in water, and 1,300 grains in spirit; what is the specific gravity of the spirit?

$$\text{Specific gravity} = \frac{2,300 - 1,300}{2,300 - 1,000} = \frac{1,000}{1,300} = \cdot 769.$$

286. **Hydrometers** are instruments for determining specific gravities, chiefly of liquids, by observing either

(1) The weights required to sink them to the *same depth* in different liquids, or

(2) The *different depths* to which they sink in the liquids.

Hence they are of two kinds: those of *constant immersion*, and those of *variable immersion.*

287. **Nicholson's Hydrometer** (Fig. 105) is an example of the hydrometer of constant immersion. It consists of a hollow copper cylinder, A, with conical ends. From the lower end is hung a small cup, sufficiently heavy to make the instrument float with its axis vertical; the upper end has a stem bearing a pan for holding weights, and having a small bead, C, called the standard point.

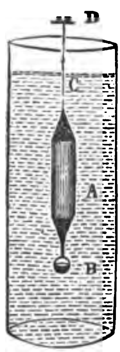


Fig. 105.

288. To determine *the specific gravity of a liquid.* Immerse the hydrometer in water. It is usually so made that when 1,000 grains are placed in the pan the instrument sinks to C. The water displaced must then weigh 1,000grs. + the weight of the hydrometer. Now immerse it in the given liquid. Suppose the weight required to sink it to C be 1,500 grains. The weight of liquid displaced is therefore equal to 1,500 grains and weight of instrument. The specific gravity of the liquid $= \frac{1500 + W}{1000 + W}$. Or, if W be the weight of the instrument, w the weight required to sink it to C in water, and w' the weight required to sink it to C in the given liquid, specific gravity $= \frac{W + w'}{W + w}$.

289. *Specific gravity of a solid.*—Immerse the hydrometer in water, and load it with 1,000 grains, it sinks to C. Place the given substance on the pan D: C will now sink below the surface: remove weights = W, till C be again at the surface. W gives the weight of the body in air. Now shift the solid to the cup B; C will rise above the surface. The weight, w, which added now, will sink C to the surface, is the weight of the water displaced by the solid.

The specific gravity is therefore $= \frac{W}{w}$.

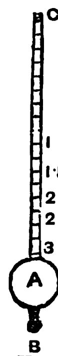


FIG. 106.

290. **Hydrometers of Variable Immersion.**—The Common Hydrometer (Fig. 106) is of this kind. It consists of a hollow glass globe, A, to which are attached—below, a smaller sphere, B, loaded with mercury, to keep the instrument vertical—above, a long graduated stem. The graduations are sometimes placed at equal distances, and sometimes denote equal differences of specific gravity. The method of graduation may perhaps be understood from the following:—

Mark the level to which the instrument sinks in distilled water at 62° Fahr., as 1. Immerse it next in a liquid whose specific gravity is already known as 2, and place 2 on the stem. In the same way immersing it in liquids of known specific gravity we obtain the positions marked 1·5, 2·5, 3, &c.; and from liquids whose specific gravity is less than that of water, we obtain the decimals to be inserted above 1. The instrument is thus graduated by being sunk in a great many liquids of known specific gravity. If afterwards we require to *find* the specific gravity of any liquid, it is only necessary to immerse it in the liquid, and observe the mark on the stem; this will at once give the specific gravity. The Common Hydrometer cannot be used to find the specific gravity of a solid. Instruments of this kind are in very extensive use for determining the density and goodness of fluids of various kinds. When used to test spirits they are called alcoholimeters; when for milk, lactometers.

291. **Sykes's Hydrometer,** which is used by the Excise Officers in finding the specific gravity of spirits, differs from the common hydrometer in having above a very thin graduated brass bar, and below a portion of stem capable of carrying one or more brass weights, instead of the mercury at B

(Fig. 106). The specific gravity of the liquid is determined, by means of a properly calculated table, from the number of weights attached at B, and the depth to which the instrument sinks in the liquid, as shown by the graduation on the stem.

292. In some cases we should have to use some other liquid instead of water, *e.g.*, to find the specific gravity of a *body soluble in water*, we may employ some liquid in which it is not soluble, and whose specific gravity is known. Weighing the body first in air, then in this liquid, the loss of weight gives us the weight of an equal bulk of this liquid. Dividing this by the specific gravity of the liquid, we get the weight of the water which the body would displace.

Example.

A piece of sodium weighs 6 grammes in air, and ·6804 gramme in naphtha of specific gravity ·86; find the specific gravity of sodium.

The weight of the naphtha displaced $= 6 - {\cdot}6804 = 5{\cdot}3196$ grammes. The weight of an equal bulk of water $= 5{\cdot}3196 \div {\cdot}86 = 6{\cdot}185$ grammes. $\therefore$ specific gravity of sodium $= 6 \div 6{\cdot}185 = {\cdot}97$.

293. Let A B C D be a bent tube of any shape, and of uniform or variable section; and let two liquids which do not mix occupy the parts A B and B C D respectively, and be in equilibrium, their common surface at B will be horizontal. Let its plane meet the other branch of the tube at C.

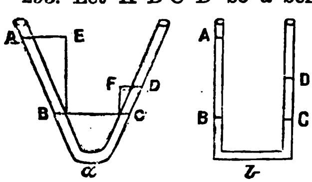


FIG. 107.

Draw vertical straight lines B E, C F, meeting the planes of the free surfaces at A and D in E and F. Let B E $= h$ inches, and C F $= h'$ inches.

Now, the pressure on a square inch at B as caused by the fluid in A B is = that at B caused by the fluid in B C D, for otherwise the surface would be moved. But the pressure caused by A B at B is = the weight of a column of the liquid of height B E, and horizontal section a square inch or to $h \times 252{\cdot}5 \times \rho$, where ρ is the density of the liquid in A B.

The pressure caused by B C D at B = pressure caused by C D at C (B and C being in the same horizontal plane) = weight

of column of *this* liquid of height C F, and horizontal area a square inch.

$$= h' \times 252{\cdot}5 \times \rho';$$

and these two pressures are equal.

$$\therefore h \times 252{\cdot}5 \times \rho = h' \times 252{\cdot}5 \times \rho'$$

$$\therefore h\rho = h'\rho'.$$

Hence, if two liquids which do not mix meet in a bent tube, the heights of their free surfaces above the level of their common surface will be inversely proportional to their densities. Thus, if the liquids are water and mercury, for every inch of mercury above the common surface, there must be 13·6 inches of water; for since mercury is 13·6 times as heavy as water, there must be 13·6 times as much water to balance it.

TABLE OF SPECIFIC GRAVITIES.

294. SOLIDS.

Standard, Distilled Water, at 62°F.

Agate	2·60	Marble	2·84
Aluminum	2·67	Mica	2·93
Antimony	6·70	Nickel	8·28
Arsenic	5·76	Nitre	1·90
Bismuth	9·88	Opium	1·34
Brass	8·30	Phosphorus	1·77
Brick	2·00	Platinum	21·47
Butter	0·94	Potassium	0·86
Chalk	2·66	Ruby	0·91
Coal	1·30	Silver	10·47
Cobalt	8·60	Sodium	0·97
Copper	8·95	Sugar	1·61
Diamond	3·52	Tin	7·29
Felspar	2·44	Wood, Ash	0·84
Glass (crown)	2·52	„ Beech	0·85
„ (flint)	2·76	„ Box	1·33
Gold	19·35	„ Cork	0·24
Granite	2·70	„ Ebony	1·21
Ice	0·92	„ Elm	0·67
Ivory	1·82	„ Mahogany	1·06
Iron	7·25	„ Oak	1·17
Lead	11·35	Zinc	7·19
Manganese	8·00		

295. LIQUIDS.

Standard, same as for Solids.

Acetic Acid	1·063	Milk of cow	1·030
Citric „	1·034	Naphtha	0·848
Muriatic „	1·200	Oil, Linseed	0·940
Nitric „	1·271	„ Olive	0·915
Sulphuric „	1·850	„ Whale	0·923
Alcohol, pure	0·797	Sea Water	1·027
„ proof spirit	0·923	Dead Sea Water	1·240
Beer	1·025	Wine, Claret	0·993
Human Blood	1·053	„ Burgundy	0·991
Sulphuric Ether	0·770	„ Port	0·997
Mercury	13·600	„ Champagne	0·997

Different specimens of the same material vary considerably in their density; the numbers in the Table are mere approximations to the average densities of the substances named.

EXERCISES.

1. Describe the *Hydrostatic Balance*, and show how to find the specific gravity of a body by means of it.

2. A body whose specific gravity is unity, weighs 6 oz. in air; what is its weight in water? *Answer.* Nothing.

3. How can you find the specific gravity of an insoluble substance?

4. A sphere weighs 3 oz. in air, its diameter being one inch; what will it weigh when wholly immersed in water? *Answer.* 1180·3 grs.

5. A body whose specific gravity is 7, weighs 18 oz. in water what is its weight in air? *Answer.* 21 oz.

6. The solid content of a body is one cubic foot, and its specific gravity is ·9. When floating in water, how many cubic inches of water will it displace? State the principle from which you deduce your answer. *Answer.* 1555·2 cubic inches.

7. A bent tube has a uniform section of a square inch; when 12 cubic inches of water have been poured into it and its straight branches are vertical, 3 cubic inches of water are in each branch. Into one branch are now poured 3 cubic inches of oil (specific gravity =·8); find how much the surface of the water in that branch is depressed. *Answer.* 1·2 inch.

8. A cup when empty weighs 6 oz.; when full of water it weighs 16 oz.; when full of petroleum it weighs 14¾ oz. What is the specific gravity of the petroleum? *Answer.* ·875.

9. Show that the resultant of the fluid pressures on a body wholly or partly immersed equals the weight of the fluid displaced.

10. Given that a pint of water weighs 20 oz., and that the specific gravity of proof spirit is ·916; what fraction of a quart of proof spirit will weigh 30 oz.? *Answer.* ·818.

CHAPTER XVII.

CAPILLARITY.

296. A tube of very small bore is called a **capillary tube** (from capillus, a hair, L). If liquid be contained in such a tube, deviations from the principles of hydrostatics are observed. The same deviations occur (but in a less marked degree) in the case of vessels containing liquid, with reference to the portion of liquid very near the sides.

297. Near the sides of a vessel the surface of the liquid is not quite horizontal. If the liquid be such that it wets the surface of the vessel, as water in a glass vessel A (Fig. 109), the surface is concave, that is, the liquid close to the side of the vessel is elevated. If the liquid does not moisten the vessel, as mercury in a glass vessel, or water in a greased vessel (B), the surface is convex, or the liquid near the side of the vessel is depressed.

298. If a very narrow tube of glass be placed in water, or any other liquid which will moisten it, the liquid will rise higher in the tube than its level outside (C). The surface of the water inside the tube is concave, and the water is elevated round the outer surface of the tube. This is called **capillary elevation** or **ascension**.

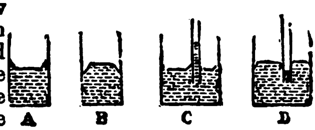


FIG. 109.

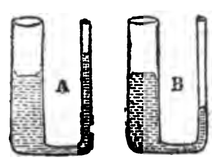


FIG. 110.

299. If, on the contrary, a glass tube be dipped in mercury, a **capillary depression** takes place, as seen at D. The surface of the mercury inside the tube is convex, and the liquid is depressed round the outer edge of the tube.

300. Let A and B (Fig. 110) be two bent glass tubes, each having one branch pretty wide and the other a capillary tube. Pour water into A and mercury into B. In A the water will

stand higher in the capillary branch, and its surface will be concave. In B the mercury will be lower in the capillary branch, and its surface will be convex.

301. The following are the observed laws of capillarity :—

(1.) *When a capillary tube is placed in a liquid, elevation will occur if the liquid moistens the tube ; if not, depression.*

(2.) *The amount of elevation varies with the nature of the liquid, and diminishes as the temperature increases.*

(3) **Law of Diameters.**—Other things being alike, capillary elevations and depressions are *inversely proportional to the diameters of the tubes.* This law was derived from the mathematical explanation of capillary phenomena, and was verified experimentally by Gay-Lussac. In a tube $\frac{1}{50}$ inch in diameter, water rises to the height of $2\frac{1}{2}$ inches ; if the bore be $\frac{1}{100}$ inch, the rise is 5 inches ; if $\frac{1}{200}$ inch, the rise is 10 inches.

302. Capillary phenomena take place equally in air and in vacuo : they do not, therefore, depend on the atmosphere. They do not depend on the substance or thickness of the tubes; for in glass tubes the amount of elevation or depression is the same for tubes of different thickness, provided their bore be the same. The real cause seems to be a certain attraction or repulsion between the molecules of the solid and those of the liquid ; and among the molecules of the liquid itself. It seems confined to a very thin film at the surface of the solid, and a similar film at the outside of the liquid.

303. The following is the usual explanations, but the subject is one of great obscurity : Let A (Fig. 111) be a plate of glass in a vessel of water, and m a molecule of water in contact with it. m is acted on by gravity m G ; by the attraction of the liquid C in the direction m 2, and by the attraction of A in the direction m n. Compounding these three forces, the resultant m R is obtained. Now the surface of a liquid at rest must be perpendicular to the force acting upon its molecules. (Art. 250.) Therefore the surface will assume the position shown at m. If these forces have very different

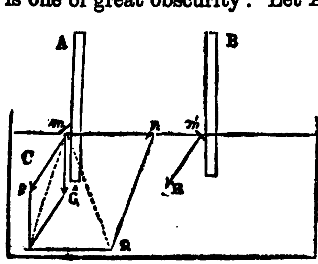


FIG. 111.

relative values, the resultant m R may assume a different direction, and the surface may be depressed, near the solid, as at m'. In general, it may be stated that if the liquid be more than twice as dense as the solid, the extreme particles of the liquid will be less attracted by the solid on one side than by the liquid on the other, and depression will occur; this is the case with mercury and glass, with water and dry cork.

304. Hold two plates of glass in water, with two vertical edges together, and inclined to each other at a small angle; the liquid will rise between them, and urge them together (Fig. 112). The height of the water at any place above the general surface is just half what it would be in a tube of diameter equal to the distance of the plates at that place. If the plates be placed similarly in mercury, a corresponding depression takes place. The section of the surface, A B, or A C, is in either case a hyperbola.

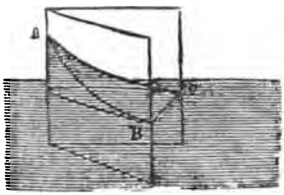


FIG. 112.

305. If a drop of water be placed in a small conical glass tube, it assumes the form represented at A (Fig. 113), and moves towards the vertex. If the drop be of mercury it will tend to move from the vertex (B). In A, the attraction between the glass and the water is greater where the diameter is less, therefore it overcomes the attraction at the wider end, and moves the drop to the smaller. In B, the repulsion at the narrow end is greater, and therefore moves the drop away from that end.

FIG. 113.

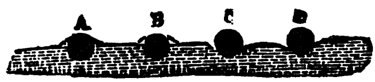


FIG. 114.

306. In Fig. 114, A and B are two small balls, floating in a liquid which moistens them. The liquid rises round them on both sides. When they are brought close, the surface between

them is raised above the general level, the balls are drawn towards each other and come into contact. Here the raised portion attracts each ball more than the lower liquid beyond them, and draws them together. If neither of the balls, C D, be moistened by the liquid, they will likewise be drawn together. The repulsion of the liquid outside them is greater than that of the lower portion between them, and hence drives them together. But when one, B, is, and the other, C, is not, moistened by the liquid, the balls when brought close repel each other.

307. Examples of the action of capillarity are very common. The oil rises through the wick of a lamp, and the melted tallow in a candle, by virtue of this force. The absorption of water by a sponge, loaf sugar, &c., and of ink by blotting paper, is another illustration. In certain vessels of plants the sap is raised by capillary attraction.

EXERCISES.

1. What are the observed laws of capillary attraction? Give examples of its action in nature.
2. If two wine glasses are taken and partly filled, one with water and the other with mercury, what difference is to be observed in the form of the surface of the liquids? Show this by a figure.

CHAPTER XVIII.

THE MOTION OF LIQUIDS.

308. **Torricelli's Theorem.**—*The velocity with which liquid issues from a small orifice in the side or bottom of a vessel is equal to the velocity which would be acquired by a body in falling from the level of the surface of the liquid to the level of the aperture.*

If a vessel like that in Fig. 115 be filled with liquid, and the tube B pointing upwards be opened, the liquid will rise in a jet nearly as high as the surface A in the vessel. It would rise quite as high were there no friction against the sides of the tube, and no resistance of the air to be overcome. Therefore the velocity of the particles at the orifice B is sufficient to raise them in opposition to gravity to the height of the surface A, *i.e.*, it is the velocity which would be acquired by falling from A, as stated in the proposition. Many experiments have been made on the subject, the result invariably being to prove the truth of the theorem.

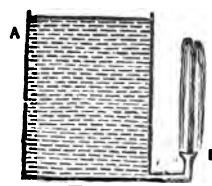


FIG. 115.

Hence the velocity is proportional to the square root of the depth. To double the velocity we require to make the depth of the orifice four times as great, for twice the quantity of matter has to be moved with twice the velocity. The flow of liquid from an orifice does not in general depend on the nature of the liquid, but only on the depth of the orifice below the surface.

If h be the depth of the orifice below the surface of the liquid, and v the velocity of the spouting fluid,

$$v^2 = 2\,g\,h \text{ and } v = \sqrt{2\,g\,h}.$$

309. If a vessel with vertical sides filled with liquid be allowed to empty itself by a small orifice in the bottom, the

quantities flowing out in successive seconds will be as 9, 7, 5, 3, 1, or as the spaces described by a falling body, in reverse order. The jet in this case is vertically downwards; a hollow forms on the surface of the liquid, which deepens into a funnel, and the particles flow towards the apex of the funnel, but rotate as they approach it. If the aperture be in the vertical side of the vessel, the jet will follow the laws of projectiles, and describe a parabolic curve. In both cases the jet continues unbroken for some distance beyond the orifice, but narrows for a distance equal to about half the diameter of the orifice. Its narrowest part is called the **vena contracta** (v. c. Fig. 116); on passing this, the jet again expands and divides, each drop holding an independent course. The amount of contraction depends on the thickness of the vessel and the size and form of the aperture. But, generally, the section at the *vena contracta* is $\frac{5}{8}$, or more accurately $\frac{62}{100}$, or ·62 of the area of the orifice. Hence the actual discharge is only ·62 of the theoretical discharge for any orifice. If a tube of a length equal to two or three times the diameter of the orifice is inserted, the discharge becomes ·82 of the theoretic amount; and if the end of the tube is made conical, and the wide end inserted in the orifice, the actual discharge becomes ·92 of the theoretic.

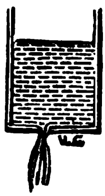


FIG. 116.

310. *The rate of efflux* of the liquid depends on the velocity and the size of the aperture. If a be the area of the orifice and v the velocity $= \sqrt{2gh}$, the rate of efflux per second will be $a\sqrt{2gh}$, and the quantity Q flowing out in any time t will be $= at\sqrt{2gh}$.

$$\therefore Q = at\sqrt{2gh} \;\therefore\; t = \frac{Q}{a\sqrt{2gh}}.$$

Thus we have the time required for any quantity, Q, of fluid to flow out, supposing the vessel to be kept full, and supposing no contraction of the jet to take place. Taking into account the effect of the *vena contracta* we obtain

$$Q = \cdot 62 \times at\sqrt{2gh} \text{ and } t = \frac{Q}{\cdot 62 \times t\sqrt{2gh}}.$$

If the liquid has to pass through tubes of different bore, which are kept full, its velocity at any part of its course will be in inverse proportion to the sectional area, *i.e.*, the velocity will be greatest in the tubes of smallest bore. Hence the

velocity of a river is greatest where it is narrowest; for the same quantity of water must pass there in a given time as at any other part, therefore the small section must be compensated for by greater velocity.

311. When liquid passes through a long tube its motion is retarded by hydraulic friction, which lessens the discharge considerably. The external portions are retarded most, and the central part moves most rapidly. The current of a river is produced by the difference between the level of its source and the level of its mouth, and the greater this difference the greater will be the velocity of the current. The central portion of a river moves quickest because the sides and under parts are retarded by friction against the bed of the river, and the surface portion in a less degree by the resistance of the air. Hence in going up a river in a vessel it is well to "hug the shore," for near the bank the water moving less rapidly offers less resistance.

312. *To find the distance to which water will spout through a small orifice in the vertical side of a vessel placed on a horizontal plane.*—Let A B C D (Fig. 117) be the vessel filled with water, and E an orifice in its side. Describe the semicircle A F B on A B, and draw the ordinate E F. Make B G = 2 E F; the water will spout from E along a parabolic curve to G. If the orifice be at H, the middle point of B C,

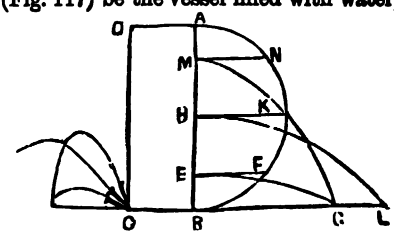


FIG. 117.

the liquid will spout to L, so that B L = 2 H K, and this is the greatest distance to which any jet will extend on a horizontal plane. If M and E be equidistant from H, above and below, M N = E F, therefore the liquid will spout to the same distance from any two orifices at equal distances, above and below, from the middle point H. If pipes are inserted at different angles of inclination at C, the water will be found to spout farthest through that which is inclined at 45°, and any two pipes at equal angles with this will throw the water to an equal distance.

313. *The Screw of Archimedes* (Fig. 118) is an instrument not now of much importance for raising water. It consists of a hollow tube wound in a spiral direction round a cylinder

with which it is turned by a handle. Water enters at the lower end; but as the screw revolves each part changes from a lower to a higher position, and the water is constantly descending with reference to the parts of the screw, while actually rising to a higher level. It is discharged at the top of the screw.

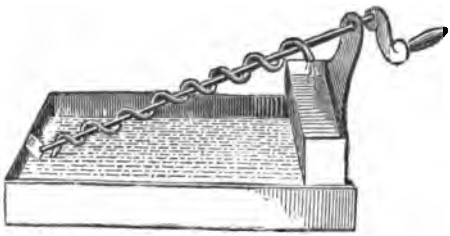


FIG. 118.

314. *The Chain Pump* consists of two wheels, A and B (Fig. 119), and a series of plates connected together by a jointed iron rod. The wheels have projecting arms which receive and support the plates. The upper wheel, turned by a winch, pulls up the chains and attached plates; these rise through a vertical tube into which they fit closely, and raise the enclosed water along with them. This pump is sometimes used in ships, and has the advantge of not being easily choked up by mud; but it is subject to great wear and tear.

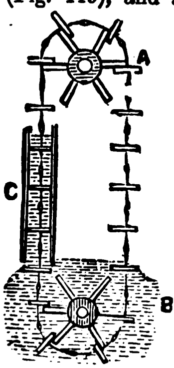


FIG. 119.

315. *The Hydraulic Belt* consists of a broad band of felt or flannel passing over two smooth wheels placed as in Fig. 119. The upper wheel being rapidly turned the belt revolves, the lower part of it being immersed in the water. A film of the liquid adheres to its surface and rises with it to the upper wheel, where it is thrown off by the centrifugal force.

316 **The Hydraulic Ram** (Fig. 120).—The fall of water from a small height produces a momentum which, by the hydraulic ram, may be utilised and made to raise a column of water to a much greater height.

A is a reservoir containing water, C is another vessel communicating at a lower level with A by the pipe B, and having two valves in its upper surface, D opening downward, and communicating with the exterior, and E opening upwards

into the vessel F. Suppose D open, water flows out through it, and more water flows down through B to supply its place. The momentum acquired by the water in this motion overcomes the weight of the valve D and closes it. This abruptly stops the flow; but the momentum is not at once destroyed. It forces up the valve E and causes the water to enter the air chamber F, until the elasticity of the air in it overcomes the decreasing momentum, and closes E. The liquid being now at rest, the weight of D overcomes the upward fluid pressure, the valve opens, and liquid flows out. This leads to a flow of water from A, and the same action takes place as before and continues as long as the cistern A is kept supplied with water. A large quantity of water is wasted, passing away through D.

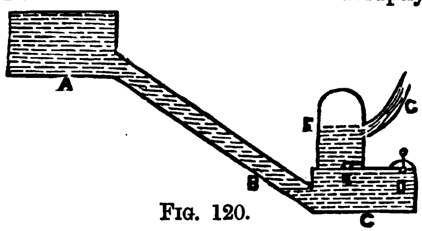


FIG. 120.

317. **The Centrifugal Pump.**—In Fig. 121, A is a vertical shaft, made to rotate rapidly by means of the handle B. C, D, &c., are pipes secured to the shaft and revolving with it, having their upper ends bent outwards, and their lower ends furnished with valves opening inwards. These pipes are filled with water, which is prevented from escaping by the valves immersed in the water. When the rotation begins, the upper ends of the pipes revolve so much faster than the lower that the water in them flies out by centrifugal force. A vacuum in the pipes is prevented by water from the cistern rushing up. The upper ends discharge the water into the circular trough, from which it passes away as required. The practical usefulness of this pump is trifling.

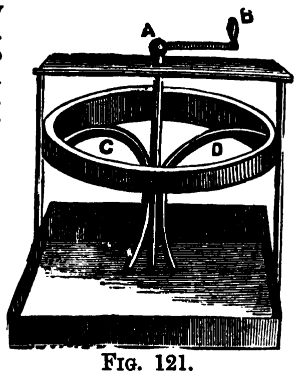


FIG. 121.

318. **Barker's Mill** (Fig. 122) consists of a vertical tube A, with two or more horizontal arms B B. The upper part of the tube with the arms can turn freely round by means of a

water-tight conical joint on the lower part. Near the end of each of the horizontal arms is an aperture on one side, all the apertures being in the same direction. Water is supplied from below by a pipe by which A is constantly kept filled.

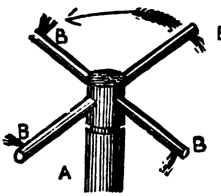


Fig. 122.

When the apertures at the ends of the arms are opened the water escapes, and pressing against the opposite sides of the arms causes a rotatory motion in the part of the tube with which the arms are connected, which is so arranged as to drive the machinery of a mill by its motion. This is thus a reaction machine. In the older forms of the machine the water was introduced from above, and its weight acting on the pivot supporting the pipe caused great loss of power by friction. This is now avoided by introducing the water from below through a water-tight joint.

319. **Water Wheels** enable us to employ the potential energy of water at a high level, or the kinetic energy of running water to produce kinetic energy in machinery. The water may act on the parts of the wheel by impact or by pressure. Two rules are received by engineers as applicable to all water wheels: (1) *The water should enter the wheel without shock*, for otherwise work is lost, and the supports or bearings of the wheel are subjected to an undue strain; and (2) *The water should leave the wheel without velocity*, for while the water has any velocity it has not parted with all the energy stored up in it. In all water wheels the greatest mechanical effect is produced when the velocity of the parts driven is half that of the stream.


Fig. 123.

Water wheels are of three kinds—overshot, undershot, and breast.

320. In the **Overshot Wheel** (Fig. 123) the water enters

the wheel near the top, and acts by its weight. This kind of wheel gives the greatest effect with the least expenditure of water, and is, therefore, applicable to mills where the supply of water is scanty. It requires a fall of rather more than the diameter of the wheel. It consists of a wheel of large radius, and generally of considerable breadth. It has a rim consisting of two annular discs, one on each side. Between these discs divisions, forming watertight troughs, extend from side to side. The water remains in these troughs through the greater part of the descent, and acting as the power at the circumference of the wheel, overcomes the resistance which is connected with the axle. The tail stream conveys away the water which falls.

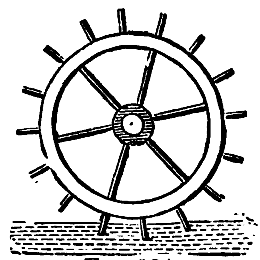


FIG. 124.

321. **The Undershot Wheel** (Fig. 124) has floats projecting from its circumference in the direction of radii. It is employed when there is no fall, but when the velocity of the stream is considerable. It is very applicable to tidal rivers, as it will turn on whichever side of the floats the current acts. This wheel has serious disadvantages. Each float on entering the water strikes it obliquely, and encounters a considerable resistance, and, on emerging, lifts a quantity of water for some distance. Both these actions lessen the efficiency of the wheel, and various contrivances have been devised for making the floats enter and leave the water without waste of power.

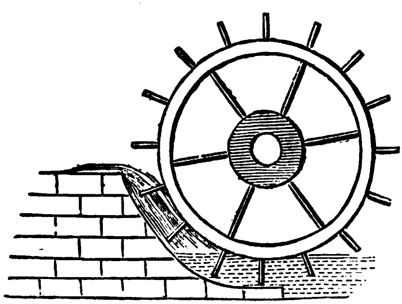


FIG. 125.

322. **The Breast Wheel** is represented in Fig. 125. The water enters the wheel somewhat below the level of the axle, and acts on the floats, which are continuations of ra[illegible] both by its weight and by the momentum due to its d[illegible]

below the level of the water in the cistern. If the floats are too far from the mason-work, a large quantity of water escapes; if too close, considerable power is lost by friction. This wheel has often troughs like those of the overshot wheel instead of floats.

323. If the dimensions, quantity of water supplied, and height of the fall be the same, an overshot will produce twice the effect of an undershot wheel; and in this comparison the breast wheel may be considered as an overshot wheel with a fall equal to the height through which the water descends. If P be the weight of water passing down in a given time, and h the height of the fall, the work available should be $= \text{P}h$. Of course part of this is employed in overcoming friction, &c., but in well constructed breast and overshot wheels 80 per cent of this work is often utilised; that is, the modulus is ·8.

324. **Turbines, or Horizontal Water Wheels,** are frequently employed on the Continent. They have large float boards placed obliquely, and these are acted on by the descending body of water in such a manner as to produce rotation in a horizontal plane. The motion is communicated to the connected machinery by the vertical axis. Turbines can be employed much better than vertical wheels, where the fall is very great. Barker's Mill is sometimes called a turbine, but it is essentially a reaction wheel, whereas the turbines described here are more properly impact wheels.

EXERCISES.

1. A vessel is filled with water to the depth of 20 feet; find the velocity with which water issues from a small orifice in the bottom. *Answer.* 35·7 feet per second.

2. A tube 60 feet high is filled with water; a small orifice opens at its bottom into a vacuum; find the velocity of the jet, the upper surface being open to the air. *Answer.* 77·56 feet per second.

3. What quantity of liquid will be discharged from a vessel 12 feet high in 1 second, through an orifice 1 inch in diameter in the bottom of the vessel? *Answer.* 162 cubic inches.

4. A cistern contains 10 feet of water, and is so fed that the water is maintained at that depth; how much water would flow through a hole at the bottom ½ inch diameter in an hour? *Answer.* 19½ cubic feet.

5. A vessel is kept filled with water to a depth of 20 feet; orifices are made at depths of 4, 10, and 16 feet; find to what distance on a horizontal plane the water will spout from each orifice. *Answer.* 16, 20, 16 feet.

PNEUMATICS.

CHAPTER XIX.

ATMOSPHERIC PRESSURE—THE BAROMETER.

325. Gases or elastic fluids differ from liquids in this respect: the particles of a liquid simply want cohesion and friction, while those of a gas have a repulsive force towards one another, and hence a gas always tends to occupy a greater space. Heat increases this repulsive force, pressure overcomes it. Hence the most marked and distinctive properties of gases are *expansibility* and *compressibility*. They are subject, in other respects, to the laws of hydrostatics. Some gases are easily converted into liquids or solids by pressure and cold; these are called *vapours*, while those which remain unchanged at ordinary temperatures and pressures are called simply *gases*.

326. The commonest and most important gas is atmospheric air. It was formerly supposed to be an element, *i.e.*, a substance not made up of different kinds of matter; but about the end of last century it was shown to be a mixture of two simple gases, oxygen and nitrogen, which are very different in their properties. Oxygen supports flame and animal life, nitrogen neither. Oxygen is somewhat heavier than nitrogen; but the two gases are always found in the same proportions. In every 100 parts by bulk of pure air, there are 79 parts of nitrogen and 21 of oxygen. The air generally contains other substances also; for instance, aqueous vapour, ammonia, and carbonic acid, but these are in small proportions. Thus, carbonic acid usually occupies about 4 parts in every 1000 by volume of atmospheric air. This complex gas may, for all the purposes of physics, be treated as a simple gas.

327. That air is an elastic fluid is shown by the following experiments:—

(1.) If a bladder *half-full* of air, and closed, be placed under a receiver, and the receiver then exhausted of air, the bladder will swell out to its full dimensions, thus proving the elasticity of the air.

(2.) If, in the air-gun, the piston is forced in, the air is compressed. If this pressure is continued, the cork is expelled; or if this is prevented by some force, when the pressure is removed the piston is forced back by the elastic force of the compressed air.

328. Though gases have a repulsive force among their own particles, they have *weight*, being attracted to the earth. The weight of the air is not felt, because at any point it is counteracted by the upward pressure, just as the hand placed flat *in water* feels no weight; but if the same quantity of water be supported *in air*, its weight is felt. Let the glass cylinder A (Fig. 126) be fitted tightly below to the plate of an air-pump: let a bladder B be tied air-tight over its mouth; then work the pump. As the exhaustion proceeds the bladder is pressed inward by the weight of the air above it, and at length bursts with a loud noise.

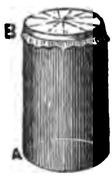


FIG. 126.

329. **Weight of Air.**—To ascertain the exact weight of air equipoise the glass globe, B (Fig. 127), by the weight A. Exhaust B by the air-pump, and again weigh it. To balance A, the weight C must be added to B, showing that B has lost part of its weight, which must be due to the removal of the air which it contained. The weight of the air removed must be equal to that of C. When the experiment is carefully made with delicate instruments, it is found that at the temperature of freezing water, and under the pressure of one atmosphere, a cubic foot of atmospheric air weighs about 1·293 ounce. A cubic foot of water weighs 1,000 ounces. Therefore air is $\frac{1\cdot293}{1000} = \frac{1}{773}$ of the weight of water. 100 cubic inches weigh 31 grs. at 60° F.

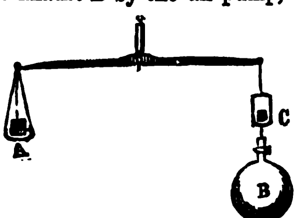


FIG. 127.

330. *The specific gravity of a gas* is the ratio of its weight to that of an equal volume of atmospheric air under the same conditions. To ascertain the specific gravity of any gas or vapour the globe in Fig. 127 is exhausted of its air and weighed; it is then filled with the given gas and weighed again, and afterwards with air, and weighed a third time. The difference between the first and second weights gives the

weight of a globe full of the given gas; the difference between the first and third is the weight of the same volume of air. The specific gravity of the gas is found by dividing the former by the latter. Thus, if the globe full of air weighs 1,250 grains, empty 250 grains, and full of ammonia 839 grains, the specific gravity of ammonia $= \frac{839-250}{1250-250} = \frac{589}{1000} = \cdot589$.

TABLE OF SPECIFIC GRAVITIES OF GASES.

Standard, Atmospheric Air, at 62° F.

Air	1·000	Nitric Oxide	1·041
Ammonia	·589	Oxygen	1.111
Carbonic Acid	1·527	Sulphuretted Hydrogen	1·180
Chlorine	2·500	Muriatic Acid	1·284
Hydrogen	·069	Steam	·622
Nitrous Oxide	1·527	Mercury Vapour	6·976
Nitrogen	·971		

FIG. 128.

331. **The atmosphere** is a shell of gas surrounding the earth to a height estimated at from 40 to 100 miles. The lower parts are densest, being pressed by the portions above. The atmosphere exerts a pressure on all bodies, just as a liquid does on bodies immersed in it. The pressure of course decreases as the density diminishes. Therefore the upward pressure on any body entirely surrounded by the air exceeds the downward by the weight of the displaced air; but this is usually so small that it may be disregarded.

332. **Torricelli's Experiment** (1643). Fill a glass tube ¼-inch diameter and a yard long, closed at one end, with mercury (Fig. 128). Place a finger on the open end, invert the tube in a cup of mercury, and remove the finger. The mercury will descend in the tube, but not to the level of that in the cup. It will remain stationary at a height of about 30 inches. The space C D will be left quite empty, and is called **Torricelli's vacuum.** It is really filled with the vapour of mercury.

333. What sustains the mercury in the tube? In other words, what prevents the free surface of the mercury in the cup from rising, so that the mercury in the tube and that in the vessel may be at the same level? No force acts on that free surface except the weight or pressure of the atmosphere. Therefore, Torricelli concluded that the pressure of the atmosphere at

any point is equal to the weight of a column of mercury about 30 inches high. If the upper part of the tube be broken the mercury in the tube will immediately sink to almost the level of that in the cup.

334. **Pascal's Experiments** (1648).—(1.) Pascal and a friend tried the Torricellian experiment at the same time, at Clermont and at the summit of Puy-de-Dôme. The mercury stood about 3 inches lower at the top of the mountain than at Clermont; thus proving that the mercury is supported by the pressure of the air, since it falls when the latter is diminished.

(2.) He tried other liquids instead of mercury in the tube. Water stood at a height of about 34 feet, or 13·6 times as high as mercury. Now mercury is 13·6 times as heavy as water; so that the two pressures are equivalent, as they ought to be.

335. **Amount of the Atmospheric Pressure.**—We have seen that the pressure or weight of the atmosphere is equal to that of a column of mercury 30 inches high. A cubic inch of mercury weighs half-a-pound; therefore the atmospheric pressure is about 15 lbs. on every square inch of surface. This is called a pressure of *one atmosphere*. The pressure of the atmosphere on the body of an ordinary-sized man amounts to 33,600 lbs. or 15 tons. That we are able to sustain this vast pressure is due to the fact that it is equal in every direction, and that all the cavities of our bodies are pervaded by air, which counteracts the external pressure. For the same reason, fishes can live at great depths in the ocean, though the pressure of the water is much greater than that of the atmosphere; in fact, this pressure is absolutely necessary for the life of these fishes. Some of them have swimming-bladders filled with air. When raised to the surface these fishes are killed by the bursting of the bladders on account of the removal of the water pressure. The effect of atmospheric pressure on bodies is not to lower, but *to raise them*. Just as in a liquid, the upward pressure on the under surface exceeds the downward pressure on the upper surface by the weight of the displaced air; therefore the resultant is an upward pressure equal to that weight. In most cases this weight is so small compared with the weight of the body that it may be disregarded. But when a body is lighter than an equal bulk of air, it ascends until it reaches a region of the atmosphere where it displaces a quantity of air whose weight is equal to its own. Balloons, smoke, and clouds are examples.

336. The Magdeburg Hemispheres show the pressure of the air in every direction. Two hollow brass hemispheres fit very closely together by their circular edges; one of them is furnished with a stop-cock. If they be placed together, and the stop-cock opened and connected with an air-pump, when the pump is worked, the air inside the globe will be drawn out, and the pressure thus removed from the inner surface. It will then require a great force to separate the two hemispheres, owi g to the uncounteracted pressure of the air on the outside. The force required may be ascertained by closing the stop-cock and suspending weights from the lower hemisphere, while the upper is held firm. In one experiment of this kind by Otto Von Güricke, the inventor of the instrument, two teams of six horses each were required to separate the hemispheres.

The upward pressure of the atmosphere may be shown in a simple manner by the *tumbler and card experiment.* Fill a tumbler with water, and cover it with a card or piece of paper; press the card close to the tumbler with one hand, and invert the tumbler with the other. The water will be supported by the pressure of the air against the card. If a bottle with a narrow neck be filled with water and inverted, the water will be supported without a card.

FIG. 129

337. The Cistern Barometer consists, in its simplest form (Fig. 129), of a glass tube like that used in Torricelli's experiment, about 33 inches long, closed at one end, filled with mercury, and inverted in a cup of mercury; the whole mounted on a secure stand. The mercury, both in the tube and the cup, has been carefully boiled, to expel air and moisture. It is evident that if the mercury in the tube sinks that in the cup will rise, though not so far; and thus the fall of the column in the tube will not give the real change in the length of the column. To remedy this an iron rod, D, with pointed ends, is attached, and may be moved up and down by a screw. One end is made to touch the surface of the mercury in B, and the distance of the other end from the top of the column in the tube is measured on the scale C. This, added to the length of the rod D, gives the height of the column, and therefore indicates the atmospheric pressure. A better plan is to have a screw by which the cistern B can be raised or

lowered, so that the mercury in the cup shall always be on a level with a fixed mark on the tube. An allowance must be made for capillary depression, and also for temperature, and for pressure of mercury vapour.

338. The barometer is used to measure the pressure of the atmosphere. This, at any place, varies at different times, and the variations indicate states of weather with some degree of accuracy. The fluctuations of the barometer, and their value as weather indications depend chiefly on the motion of the air in cyclones. It is not so much the absolute height of the barometer, as its steady rise or fall, which can be regarded as a weather indication. From this use of the barometer it is called a *weather-glass*. The height of the column varies from 28 to 31 inches, or ranges through 3 inches, and this is the length of the graduated scale. The pressure also varies, at the same time, at places of different elevations; and hence the barometer is used for measuring heights. To make barometric observations more uniform, they are generally reduced, by computation, to the sea-level. Roughly speaking, the mercury in the tube descends $\frac{1}{10}$ inch for every 95 feet ascended, up to an elevation of 3,000 feet, but various corrections are necessary on account of changes of temperature and other causes.

339. **The Wheel Barometer** (Fig. 130) is the form usually employed as a weather-glass. It is in the form of a siphon with the longer arm closed. If the surface at A sinks one inch, evidently the surface at B will rise an inch, and the difference of level will be lessened by two inches. A small change in the pressure is therefore difficult to be observed, since the fall in the column A is only half the change of level. This defect is sometimes remedied by expanding the upper part of the longer arm into a globe, so that a slight fall there will produce a comparatively large rise at B. A small ball of iron or glass floats at B, and is connected by a thread passing over a pulley, C, with a similar ball which balances it. When the atmospheric pressure diminishes, the surface at A falls and that at B rises; the ball at B is raised, and the thread passing round the pulley moves the pulley and the index attached to it. This index points to a graduated circle, and indicates very accurately the changes of pressure.

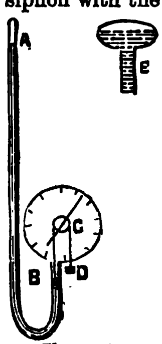


FIG. 130.

If the surface at A falls half-an-inch, that at B rises half-an-inch; this indicates a difference in the real height of the column of an inch. If the circumferance of the pulley be $1\frac{1}{2}$ inches, this rise will cause the pulley and index to revolve through 120°. In the same way, a change of $\frac{1}{2}$ an inch would produce a motion of the index through 60°; of $\frac{1}{12}$ inch through 10°; of $\frac{1}{120}$ inch through 1°. And as the index is generally made of considerable length, very small variations can thus be easily observed.

340. **The Aneroid Barometer** (*α*, without; *νηρὸς*, moisture). This barometer depends on the changes in form of a thin metallic vessel partially exhausted. As the atmospheric pressure becomes less, the vessel enlarges in one direction, and acts on a lever, which moves an index; this index points on a dial to a number indicating the corresponding pressure. The aneroid barometer is very sensitive, much less liable to injury than the mercurial, and very portable, being often so small that it can be carried in the waistcoat pocket like a watch.

341. **The Vernier, or Nonius,** (Fig. 131) is a small sliding scale attached to the side of the fixed scale of a barometer for the purpose of measuring hundredths of inches, the ordinary scale giving only tenths. It is $1\frac{1}{10}$ inch in length, and is divided into 10 equal parts; each part, therefore, is $\frac{11}{100}$ inch in length. Now each division on the fixed scale is $\frac{1}{10}$ or $\frac{10}{100}$ inch; therefore a division of the vernier is $\frac{1}{100}$ inch longer than one of the fixed scale. The use of the vernier will be understood from the example in the diagram. Here the top of the vernier is brought on a level with the surface of the mercury in the tube, and it is found that the line marked 2 on the vernier coincides with 29·5 on the scale; the next two lines do not coincide, the line 1 on the vernier being evidently $\frac{1}{100}$ inch above 29·6; so the top of the vernier is $\frac{2}{100}$ inch above 29·7. The true height of the column is therefore not 29·7, which is the nearest given by the fixed scale, but $\frac{2}{100}$ or ·02 inch more, *i.e.*, 29·72.

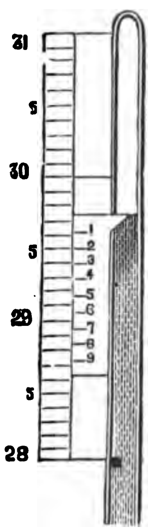


FIG. 131.

The vernier thus enables us to measure the height of the

column true to $\frac{1}{100}$ inch. Of course the vernier may easily be adapted to measure small fractions of the decimetre or centimetre.

342. Mercury is the most convenient fluid to employ in making barometers from its great weight, and the consequent shortness of the column required; but water barometers have sometimes been made. A water barometer was constructed for the Royal Society by Daniell; its tube was 1 inch in diameter and 40 feet long. The water is protected from evaporation at the open end by a solution of caoutchouc in naphtha. The average height of the column is 400 inches, but it seems to be subject to almost constant fluctuation. Very slight alterations of pressure are indicated by it, and its motions are less sluggish than those of the mercurial barometer.

343. **Height of the Homogeneous Atmosphere.** If the atmosphere were of uniform density throughout, we could easily find the height to which it would reach. We know that it supports on an average a column of mercury 30 inches high; therefore the weight of a column of air of the height of the atmosphere is equal to that of a column of mercury 30 inches high, or to a column of water $30 \times 13{\cdot}6$ inches or $5 \times 6{\cdot}8$ feet $= 34$ feet of water. Now water is 773 times as weighty as air, therefore a column of 34 feet of water is equal in weight to a column of air 34×773 feet high $= 4{\cdot}97$ miles. This is, therefore, the height of the homogeneous atmosphere. Calling it h, we see that—

$$h = \text{height of barometer} \times \text{specific gravity of mercury} \times 773$$
$$= 5 \text{ miles nearly.}$$

* 344. **Greatest Possible Height of the Atmosphere.**—Since air is a compressible fluid it is denser in its lower parts, and becomes less and less dense as the distance from the earth increases. It cannot, however, be infinite in extent, for every particle is acted on by two forces—gravity pulling it down to the earth, and centrifugal force urging it away from the earth. At the surface of the earth the force of gravity preponderates; as we ascend gravity diminishes, and centrifugal force increases. There must be some height at which these forces exactly balance each other; beyond that height centrifugal force predominates, and drives away any particles of air which are found there. This height must, therefore, be the utmost limit of the atmosphere. We shall now show how to determine this height.

The centrifugal acceleration on the surface of the earth at the equator is $f = \frac{g}{289}$

$$\text{For } f = \frac{v^2}{r} \text{ (Art. 192).}$$

$$v^2 = \left(\frac{24900}{24 \times 60 \times 60}\right)^2 = \left(\frac{249}{24 \times 36}\right)^2 = \left(\frac{83}{288}\right)^2 \text{ miles}$$

$$= \frac{36373920}{288^2} \text{ feet; and } r = 3963 \text{ miles.}$$

$$\text{Therefore } f = \frac{36373920}{288^2 \times 3963} = \frac{32}{289} = \frac{g}{289}.$$

Let f^1 be the centrifugal acceleration on a particle at any height x above the surface. Since the times of revolution are the same, the contrifugal acceleration will be proportional to the radii (Art. 193).

Therefore $f^1 : f :: r + x : r \therefore f^1 = f\frac{r+x}{r}$.

But we have found that $f = \frac{g}{289}$,

$$\therefore f^1 = g\,\frac{r+x}{289r}.$$

If now g^1 be the acceleration of gravity at the height x, since gravity varies inversely as the square of the distance,

$$g^1 : g :: \frac{1}{(r+x)^2} : \frac{1}{r^2} \therefore g^1 = g\left(\frac{r}{r+x}\right)^2$$

Now if x be the height at which we have supposed centrifugal acceleration and gravity to be equal, $f^1 = g^1$,

$$\therefore g\,\frac{r+x}{289r} = g\left(\frac{r}{r+x}\right)^2 \therefore \frac{(r+x)^3}{r^3} = 289 \therefore \frac{r+x}{r} = \sqrt[3]{289}.$$

$\therefore x = r\left(\sqrt[3]{289} - 1\right) = 5{\cdot}61r = 5{\cdot}61 \times 3963 =$ about 22,000 miles, the greatest possible height to which the atmosphere can extend.

*345. **Measurement of Heights.**—The barometer, as already stated, is often employed in measuring heights. The density of the atmosphere decreases as the square of the distance from the earth increases. It is found that at a height of $3\frac{1}{2}$ miles above the sea-level the half of the atmosphere is passed, and at that height accordingly the barometer will stand at about 15 inches. At a height of seven miles the density is only $\frac{1}{4}$ what it is at the sea-level, and the

mercury would there sink to about 7½ inches; at a height of 10½ miles the density is ⅛, and the height of the barometer 3¾ inches. Thus, while the distance increases in an arithmetical series, the density decreases in a geometrical series.

Various rules have been framed by scientific men for ascertaining heights by the barometer, but these can only at best give approximations to the truth. The following rule by Leslie is very simple, and gives pretty accurate results for heights under 5,000 feet: Note the exact barometric pressure at the base and at the summit of the elevation, then say, As the sum of the two pressures is to their difference, so is the number 52,000 to the required height in feet. Thus, if the pressures at the base and summit of a mountain be 29·52 and 25·47—

As $29{\cdot}52 + 25{\cdot}47 : 29{\cdot}52 - 25{\cdot}47 :: 52000$ feet : 3829·8 feet, the height of the mountain.

The following formula, which is sometimes used, takes into account the difference of temperature, and gives the approximate height in feet:—

$$h = 60360\left[1 + \frac{T_1 + T_2 - 64}{2} \times \frac{\cdot 3665}{180}\right] \times \log.\frac{H^1}{H^2},$$

where H_1 T_1 refer to the lower station, and H_1 H_2 are heights of the barometer; or more generally

$$h = 60360\left[1 + a \times \frac{T_1 + T_2 - 64}{2}\right] \log.\frac{H^1}{H^2},$$

where the value of a will be $\frac{\cdot 3665}{180}$ or $\frac{\cdot 3665}{100}$ according as we take Fahrenheit or Centigrade measurement of T_1 T_2, and the 64 will be omitted for Centigrade—the multiplier of a being the mean number of degrees above freezing point. Air expands from 1 to 1·3665 between 32° and 212° F., or between 0° and 100° C.; hence

$\frac{\cdot 3665}{180}$ is the expansion per degree Fahrenheit, $\frac{\cdot 3665}{100}$ the expansion for every rise of 1° Centigrade.

EXERCISES.

1. What is the difference between *elastic* and *non-elastic* fluids? Describe an experiment to show that in elastic fluids the pressure is proportional to the density.

2. How is the pressure of the atmosphere determined? Describe a common barometer.

3. How is the barometer used in calculating heights above the sea-level?

4. Show that the column of mercury in the barometer measures the weight of the atmosphere.

5. Does the apparent weight of a body alter with the pressure of the atmosphere? In what cases will this be of any importance?

6. What does a barometer measure?

7. A mercurial barometer stands at 77·4 centimetres; what would be the height of an oil barometer at the same place, the specific gravity of mercury being 13·596, and that of oil 0·9? *Answer.* 1169·25 centimetres.

*8. At the higher of two stations, the barometric height is 24·78 inches, temperature 48° F.; at the lower station, the barometric height is 29·93, temperature 73° F.; find the difference in height of the two stations, given log. 2·993 = ·60193

log. 2·478 = ·39410

constant of formula 60360 feet. *Answer:* 13272·5604 feet.

CHAPTER XX.

LAWS OF ELASTIC FLUIDS.

346. If a quantity of gas be enclosed in a vessel it will press against the sides by its weight, like any other fluid; but, besides this, it will exert a much greater pressure arising from the mutual repulsion among its particles. The latter pressure is called the *elastic force* of the gas, and is found to vary with the space the gas occupies and also with the temperature.

FIG. 132.

Let A B (Fig. 132) be a vertical tube, whose internal section is one square inch, and closed at the lower end. Introduce the piston C, fitting air-tight into the tube at A. The pressure on C from within is now one atmosphere (neglecting its own weight). Load C with a weight of 15 lbs.; it will descend to D so as to compress the air into half its original volume. Add 15 lbs. more; the piston will descend to E, so that the air occupies only one-third the original space. So if another 15 lbs. be added, the space occupied by the air will be one-quarter of A B. We thus see that doubling the pressure compresses the air into half the space, trebling it into one-third the space. Hence:

347. "*If the temperature be kept the same, the pressure (or elastic force) of a quantity of air or gas is inversely proportional to the space it occupies.*"

This law was discovered in England by Boyle, and in France by Marriotte, about the same time. It is called sometimes **Boyle's Law**, sometimes **Marriotte's Law**. Those two philosophers pursued the same method of investigation, and used similar apparatus. We shall state briefly the method by which the law is proved.

Let A B be a glass tube of uniform bore, bent into the form shown in Fig. 133, with its two arms parallel, the longer one being open, and the shorter closed by a stop-cock. Along each arm is fixed a scale, marked with inches and parts of inches. The scale B D is graduated downwards from B, the scale A C upwards from C. The stop-cock at B being open, mercury is poured in at A until it stands at the

same level C D in both arms. The stop-cock at B is now closed. Each of the surfaces of the mercury at C and D is now subjected to a pressure of one atmosphere, or 15 lbs. per square inch. Mercury is then poured in until the air B D is compressed into half the space, B F. The mercury in A will be found to be higher than that in B by exactly the height of the barometric column, 30 inches or so. Let mercury be again poured in until the air is compressed into one-third of its original volume, B K. The difference of level in the two arms is now equal to twice the barometric column. And thus the volume was found always to be inversely proportional to the pressure, through a wide range of experiments made by Boyle and also by Marriotte. The density also varies inversely as the volume; therefore the pressure and density are directly proportional to each other; and the product of volume and pressure, or of volume and density, is constant.

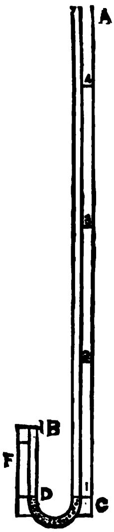


FIG. 133.

348. The same law can be shown to hold good when the volume is increased. Let the graduated tube B C (Fig. 134), partially filled with mercury, be inverted in the vessel D, containing mercury, so that the mercury stands at the same level in the tube as in the vessel. Mark the volume occupied by the air, and raise the tube. The gas expands, its pressure diminishes. The pressure on the mercury in D is greater than on that in the tube; therefore the latter rises. If the space occupied by the air be double what it was at first, the height of the mercury is half that of the barometric column; if the volume of the air becomes three times what it was, the height of the mercury is $\frac{2}{3}$ that of the barometer, *i.e.*, the pressure of the air is now only $\frac{1}{3}$ what it was at first. Or, generally, the pressure is inversely proportional to the volume.

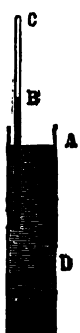


FIG. 134.

349. **Unequal Compressibility of various Gases.**—Later experiments have shown that this law of Boyle is not strictly true for all gases. This may be shown by the apparatus represented in Fig. 135. A is a strong cast-iron vessel

containing mercury, B, and oil, C ; the plunger, P, wrought by a screw, S, dips into the vessel. A is connected with another iron vessel, D, into which are firmly fixed tubes, T' T, each 6 feet long, and $\frac{1}{16}$ inch in diameter. Equal volumes of two perfectly dry gases are introduced at the upper parts of the tubes, which are then hermetically sealed. Of course the mercury now stands at the same height in both the tubes. The plunger is now screwed down, and the mercury forced up the tubes. The same pressure is evidently applied to the two gases; but it is found that the height of the mercury does not remain the same in both tubes. When great pressure is applied, no two gases follow exactly the same law.

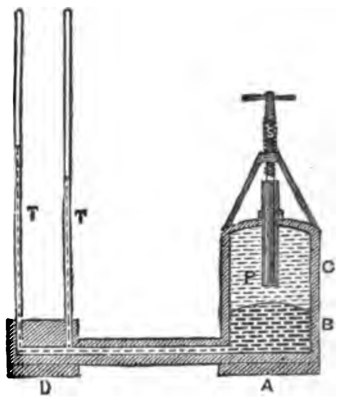


FIG. 135.

350. **Limitation of Boyle's Law.**—The result of the most recent experiments, as indicated in the preceding paragraphs, is that the law holds good, with only slight variations, for those gases which are not liquefiable, as air, oxygen, hydrogen. nitrogen. Those gases which are capable of liquefaction* exhibit considerable deviations, and the deviations are greater as the gas approaches the liquefying point. Carbonic acid, sulphuretted hydrogen, cyanogen, ammonia, are examples.

351. **Manometers** are instruments for measuring the elastic force of gases or vapours. This elastic force, pressure, or tension is expressed in atmospheres. One atmosphere indicates a pressure of about 15 lbs. on the square inch, more or less, according to the height of the barometer. Pressure is sometimes measured by the weight of a column of mercury; but this is inconvenient when the pressure is very great. To measure a pressure of 10 atmospheres, for example, we should require a column of mercury 300 inches long, and of course a tube of the same length.

352. **The Compressed Air Manometer** (Fig. 136) consists of a bent glass tube, A B, closed at one end, A, and communicating at the other end with a pipe connected with

* This is now known to be the case with all gases.

the vessel containing the gas whose pressure is to be ascertained. A quantity of mercury, C D, is placed in the tube and stands in both arms at the same level, marked 1, each surface being at first subject to a pressure of one atmosphere.

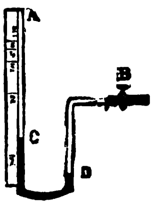


FIG. 136.

When the cock B is turned the compressed gas flows in, and increasing the pressure, raises the surface C. If the pressure be that of two atmospheres, the air will be compressed to nearly half its original volume, not quite, for the pressure, besides condensing the air, has to sustain a column of mercury equal to the difference of C and D. The number 2 is therefore marked nearly half way up. So the numbers 3, 4, 5, &c., indicate pressures of 3, 4, 5, &c., atmospheres. The marks on the scale soon approach each other very closely, so that the measurement is not very delicate. To obviate this, the tube, C A, is sometimes made conical; the graduations are then more nearly equal.

Having thus graduated the instrument, by introducing gases at known pressures of 2, 3, 4, 5, &c. atmospheres, and marked the surface positions, we can afterwards learn the pressure of any gas by observing to what height it raises the mercury in AC.

353. **Dalton's Law.**—*If a gas be heated under constant pressure, equal increments of temperature will produce equal increments of volume.*

All gases under the same pressure expand equally for equal increments of temperature; and the degree of expansion is the same for all. Many careful experiments on the subject give the following results: One volume of gas when heated under constant pressure will expand $\frac{1}{490}$ of its volume at the freezing point for every degree Fahrenheit, or $\frac{1}{273}$ of its volume at the freezing point for every degree centigrade. The quantities $\frac{1}{490}$ and $\frac{1}{273}$ are called *coefficients of expansion.*

Thus 490 cubic inches of any gas at 32° become
491 " " at 33 degrees.
492 " " " 34 "
500 " " " 42 "
$490+t$ " " " $32+t$ "

And 273 litres of any gas at 0° C. become
274 " at 1° C.
275 " " 2° C.
283 " " 10° C.
$273+t$ " " t° C.

Examples.

(1) 524 cubic inches of air are heated from 32° to 56° under constant pressure. Find what value they occupy.

As 490 : 490 + 24 : : 524 : 549·6 cubic inches.

(2) 312 cubic inches of air are cooled under constant pressure from 85° to 47°. Find the volume at the lower temperature.

490 + 53 : 490 + 15 : : 312 : 290·2 cubic inches.

(3) 350 litres of carbonic acid gas are heated from 17° C. to 84° C. Find the volume.

273 + 17 : 273 + 84 : : 350 : 430·9 litres.

(4) 500 litres of gas at 65° C. are cooled down to 15° C. Required the volume.

273 + 65 : 273 + 15 : : 500 : 426 litres.

354. **Mixture of Gases.**—*When two gases which do not chemically unite are mixed together each gas acts as a vacuum with respect to the other.*

If two liquids of different densities are placed in the same vessel the heavier will remain at the bottom, the lighter will float on the top. But gases behave in quite a different way. If a jar full of carbonic acid ($1\frac{1}{2}$ times as heavy as air) be placed mouth upwards on a table, and another jar of the same size full o. hydrogen ($\frac{1}{14\frac{1}{4}}$ part of weight of air) be placed mouth downward close over it, it will be found after some time that the carbonic acid gas has risen and diffused itself equally through the two jars, and that the hydrogen has descended and diffused itself in the same manner; so that each jar is filled with a mixture in equal proportions of the two gases. The diffusion of either of the gases through the space occupied by the other does not take place in the same time as if there had been a vacuum, but the ultimate result is the same.

355. **Kinetic or Dynamic Theory of Gases.** — Many circumstances lead to the belief that the molecules of every body are in constant motion among themselves; they seem to undulate back and forward in straight lines. Heat increases the activity and enlarges the swing of these undulations. In solid bodies the molecules, though in continual motion, do not pass to a sensible distance from their original position. In a fluid the molecules are constantly coming into collision with one another; but there is nothing to determine a molecule to return to its original place rather than to travel in another direction. The molecules of a gas move in straight lines with uniform velocity. When two molecules come into collision

each has its course changed, and starts on a new path with an altered velocity. This collision is called the *encounter* between the two molecules, and the path traversed between one encounter and another the *free path*. As the density of a gas increases the free path diminishes, and in liquids the molecules are so closely aggregated that there is no free path.

The particles of a gas, then, are like so many small bullets playing incessantly back and forward and impinging against each other and against the boundaries by which the gas is enclosed. Thus a gas exercises a constant pressure against the sides of a vessel in which it is contained. If the size of the vessel is increased the gas still fills it and presses against its sides; but the pressure is diminished, because fewer particles strike against a given area in the same time. If the gas is heated the pressure is increased, for the velocity of the molecules is increased by heat, and hence a greater number of molecules moving with a greater velocity strike against a given area in the same time. This theory, which has been most fully developed by Clausius, accounts satisfactorily for the laws of elastic fluids. (See Clerk Maxwell, "Theory of Heat.")

EXERCISES.

1. State and explain clearly the law which connects the pressure, density, and temperature of an elastic fluid.

2. A cubical vessel, whose edge is ten inches, is compressed into one whose edge is four inches; supposing it to have been hollow and filled with air, how much is the internal pressure on one side of the cube increased? *Answer.* $2\frac{1}{2}$ times.

3. A given quantity of elastic fluid occupies different volumes under different pressures. What is the law which connects the volumes and the corresponding pressures, assuming the temperature to remain unchanged? A bent tube of uniform bore, closed at one end, has the air in the closed end isolated by mercury, which fills the bend. When the height of the barometer is $28\frac{1}{2}$ inches, the mercury stands at the same level in the two legs, and the enclosed air occupies 8 inches. On the barometer rising the difference of level of the mercury in the two legs is $\frac{1}{2}$ inch; what is now the height of the barometer? *Answer.* 29·92 inches.

4. A barometer tube has an internal section of $\frac{1}{2}$ of a square inch. There are 30 inches of mercury standing in it, and above it 6 inches of vacuum. A bubble of air containing $\frac{1}{10}$ cubic inch of external air is introduced. Show that (neglecting changes of temperature) the mercury will fall 4 inches in the tube.

5. Describe some simple experiment to show that the pressure of air is increased by heat.

6. The two limbs of a Mariotte's tube are graduated in inches. The mercury in the shorter limb stands at graduation 4, and five inches of air are inclosed above it. The mercury in the other limb stands at the graduation 38. The barometer at the time indicates a pressure of 29·5 inches. Find to what pressure the five inches of air are subjected, and also the length of tube which they would occupy under the atmospheric pressure alone. *Answer.* 2·153 atmospheres; 10·765 inches.

CHAPTER XXI.

INSTRUMENTS DEPENDING ON THE PROPERTIES OF GASES.

356. If a wine glass be inverted and immersed mouth downwards in water, it will be observed that the air inside the glass is compressed by the upward pressure of the water, and that, though the water rises within the glass, it does not rise so high as the level of the water outside. The space occupied by the air will be inversely proportional to the pressure; so that if the pressure, including that of the atmosphere, be doubled, the air will occupy only one-half of its original volume. This experiment illustrates the principle of the **Diving Bell**, which is a strong iron vessel, either cylindrical or bell shaped (Fig. 137), open at the bottom, and containing seats for two or more persons. Its weight exceeds that of the water it would contain. When lowered by a chain the air within it is compressed, but prevents the water from rising high in the bell, so that persons can descend in it to a considerable depth. Light is admitted by thick glass plates inserted in the upper part. At a depth of 32 feet the bell would be half filled with water, and the compression of the air would increase with the depth. To prevent the water from rising too high, fresh air is pumped down through a flexible tube, opening under the mouth of the bell; and the vitiated air is allowed to escape by opening the stop-cock B. By means of this instrument bodies and goods are recovered from sunken vessels, and the foundations of piers, bridges, and lighthouses are constructed under water.

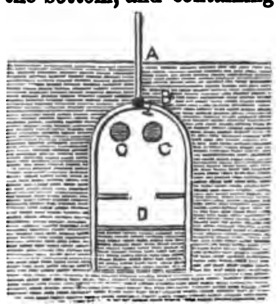


FIG. 137.

The air within the vessel is subjected to a pressure composed of the atmospheric pressure and that of a column of water whose height is the depth of the surface of the water inside the bell below the surface of that which is outside. Thus, if this depth be 45 feet, the air is subjected to a pressure of $34 + 45 = 79$ feet of water $= 79{,}000$ ounces, or 4937·5 lbs. per sq. foot.

To find the space occupied by the air at any depth (supposing no fresh air forced in) :—

Since the volume is, by Boyle's law, inversely as the pressure, and the pressure at the surface is equal to a column of 34 feet of water, we have, calling the content of the bell S :—

$$\text{Vol. at any depth } h : S :: 34 : 34 + h.$$

$$\therefore \text{Vol. at any depth } h = \frac{S \times 34}{34 + h}.$$

Thus, if a cylindrical diving-bell 12 feet high be sunk to a depth of 75 feet, find to what height the water will rise.

Here, the space occupied by the air at a depth of 75 feet

$$= \frac{12 \times 34}{34 + 75} = \frac{408}{109} = 3{\cdot}744 \text{ feet.}$$

Therefore the water will rise 8·256 feet.

Of course the surface pressure is not always equal to 34 feet of water; but to the height of barometer $\times$ 13·6.

357. Though pumps for raising water were used in very early times, their principle was not understood till the 16th century, when Torricelli made his experiments, already mentioned (Art. 332). The construction and action of pumps are easily understood after the description of the barometer. If the atmospheric pressure is removed from any portion of the surface of a liquid, the liquid will rise there to a height depending on its weight. Thus, if we place one end of a straw in the mouth and the other in water and draw in the air, the water will rise. If our straw were 30 feet long we could still raise the water; but if its length exceeded 34 feet, the water would not reach quite to the top of it. The old explanation of this was that *nature abhors a vacuum*, and therefore when the air is removed, fills up the place with water. But the ascent of the water is now known to be due to the pressure of the atmosphere.

358. **The Common Suction Pump** (Fig. 138.) consists of a cylinder of wood or metal, A, joined below to a smaller tube,

B, called the *suction-tube*, which dips into the water. At the junction of A and B is a valve,* C, opening upwards, and in the piston, P, is another valve, D, also opening upwards. When the piston is forced down by working the pump-handle, the valve D opens in consequence of the increased density and therefore pressure of the air under it. When it has descended to the bottom of the cylinder, let it be again raised. The air in the cylinder being prevented from filling the lower part by the valve D closing, that in B will open the valve C and expand. The pressure on the water is lessened; it therefore rises in the tube to a certain distance. The piston again descends, C closes, D opens. On the next ascent of P the air is still further rarefied, the water rises higher in the tube, and at length enters the cylinder A. At the next stroke it passes through D, and flows out at E. While the handle is worked, a constant flow of water is maintained from F.

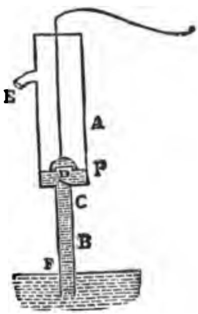


FIG. 138.

359. **Limits of its Action.**—Since the atmospheric pressure can support, at most, a column of water about 34 feet high, if C be more than 34 feet above the surface of the reservoir the pump cannot act at all. The suction-tube, B, is, therefore, rarely more than about 26 feet long. If the highest position of the piston be more than 34 feet above the reservoir, the flow of water from the spout will not be constant. The spout may be at a considerable distance above C, the water being lifted by the ascending piston; but the farther it is lifted the more difficult will it be to work the pump.

360. **Force required to Work it.**—Suppose the piston in the position shown in Fig. 138, and with water flowing out at the spout. Every square inch of the upper surface is subject to the atmospheric pressure + weight of a column of water, of height E D, *i.e.*, to $a + w.$ E D ($w.$ = weight of a cubic inch of water).

Every square inch of the lower surface is under a pressure of an atmosphere—weight of a column of liquid, of height D F, *i.e.*, $a - w.$ D F.

* *A valve* is a lid attached to any opening in a tube or piston in such a way that it can open only in one direction.

The resultant pressure will be the difference of these pressures.

$$(a + w.\ E\,D) - (a - w.\ D\,F) = w.\ (E\,D + D\,F) = w.\ E\,F.$$

That is, the resultant pressure on each square inch of the piston is the weight of a column of water one square inch in section, and of height E F. This pressure is the same for every position of the piston. Therefore *the force required to work the pump* will be the weight of a column of water, whose bore is equal to the area of the piston, and height, to the height of the spout above the surface of the water.

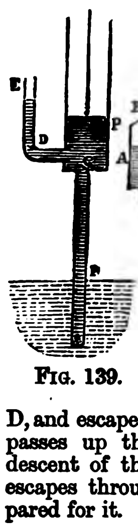


FIG. 139.

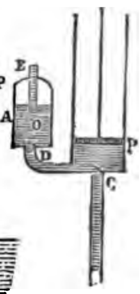


FIG. 140.

361. **Efficiency of the Pump.**—Neglecting friction, which considerably diminishes the effect of a pump, the force required to raise the piston $\times$ space through which the piston is raised = weight of water discharged $\times$ height to which it is raised or, $f.\ E\,C = w.\ E\,F$.

362. **The Forcing Pump.**—In this pump (Fig. 139) the piston is solid. When the piston descends, the compressed air opens the valve D, and escapes by the pipe E. After a few strokes the water passes up the suction-tube F C into the barrel. By the descent of the piston the valve C is closed, and the water escapes through D into E, and passes into the cistern prepared for it.

363. Another form of Force Pump is shown in FIG. 140. Here the water is forced through D into an air-tight vessel A, into which the pipe E dips. When the water rises above O, the air in the upper part of A is compressed, and by its pressure on the water forces it up through E in a continuous stream.

The Fire Engine is a double force-pump by which water is forced into an air-chamber like A. The compressed air acts powerfully on the water, and forces it out in a continuous jet through a tube to which is attached the hose, by which the water is directed to the particular spot where it is required.

364. **The Siphon** (Fig. 141) is a bent tube, having one leg longer than the other, and open at both ends. It is used for drawing off liquids from vessels. The tube is filled with liquid, the longer leg is closed with a finger, and the shorter is placed with its end immersed in the liquid in the vessel. When the finger is removed from C, the liquid flows steadily through the siphon, so long as its shorter end is under the surface.

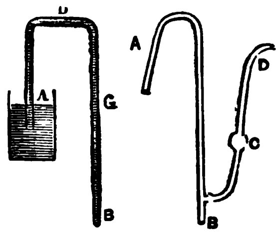


FIG. 141. FIG. 142.

365. We may explain the action in this way. While A D is not more than about 32 feet, atmospheric pressure will support the liquid in the arm A D. Now the surface B is pressed downwards by the weight of the column D B (w), and upwards by the usual atmospheric pressure (a). The resultant of these two is an upward pressure of $a-w$. At A there is an upward pressure equal to that of the atmosphere, and a downward equal to the weight of A D (w_2). The resultant is an upward pressure of $a-w_2$. If the pressures on the two ends were equal, the liquid in the siphon would be in equilibrium. But the weight of D B is greater than that of D A, therefore $a-w$ is less than $a-w_2$. The liquid will therefore be urged in the direction in which the latter acts, viz., from A to B. The force urging the liquid is equal to the difference of the pressures $=(a-w_2)-(a-w)=w-w_2$, *i.e.*, the weight of the column G B.

366. A more convenient form of siphon is shown in Fig. 142. A tube D is attached to the longer leg. When the siphon is to be used, the shorter leg A is dipped in the liquid, and the mouth applied at D to suck out the air, B being closed by a finger. The bulb C prevents the liquid from rising easily to the mouth; hence this form is employed in drawing off liquids of a disagreeable or dangerous nature. As soon as the liquid has reached the bulb, the mouth may be withdrawn.

367. **Intermitting Springs** may be explained on the principle of the siphon. Thus, if A (Fig. 143) be a cavity in the interior of a hill, with cracks or passages B, C, &c., through which water trickles, so as to fill the reservoir to a certain height,

the water will also rise in D F to the same level. As soon as the surface reaches the level of the passage at F, it will flow through, and by the principle just explained, will continue to flow until the water in A sinks to the level of D, when the spring will cease and no more water will flow out at G until a new supply finds its way to A, through B, C, &c. Thus the spring is intermittent in its action.

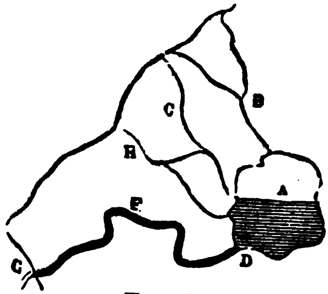


FIG 143.

368. **The Syringe or Condenser** (Fig. 144).—A is a tube having an air-tight piston moving up and down in it. The piston contains a valve, D, opening downwards; another valve, C, at the end of the barrel, also opening downwards, communicates with the glass globe screwed on below C. P, moving down, condenses the air between P and C. D closes, C opens, and air is forced into B. When P rises C is closed by the increased elastic force of the air in B, and D opens. Thus air is admitted between P and C. At its next descent P condenses this air, and drives it into B; and thus the condensation goes on until the elasticity of the compressed air in P C is comparatively too feeble to open the valve C.

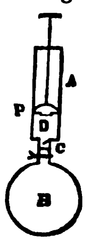


FIG. 144.

369. **The Air-Pump** was invented by Otto von Guericke, Burgomaster of Magdeburg, in 1650. It is an instrument for pumping out or rather for rarefying the air contained in a given space, and depends on the principle that gases always expand when the pressure they are under is lessened.

370. In its simplest form it is merely an exhausting syringe (Fig 145). When the piston, P, is pressed down, air passes upwards through the valve in it. When it is raised, the valve C is forced open by the elastic force of the air in D. At the next descent of B, the air in P C is compressed, the valve C is closed, the valve in P opens, and more air escapes. P again rises, C opens; and thus the action goes on

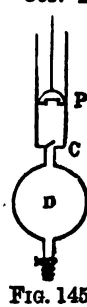
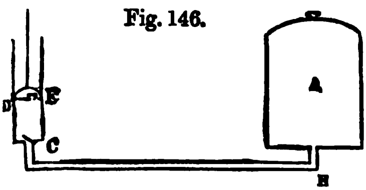


FIG. 145.

until the pressure within D is so little greater than that of the air in P C when most rarefied, that it is insufficient to open the valve C. The action of the pump then ceases, and the exhaustion is said to be complete.

371. **Smeaton's, or the Single-barrel Air-Pump** (Fig. 146). F is a piston moving air-tight along the strong brass cylinder, D. It contains a valve opening upwards. Another valve, C, also opening upwards, covers the end of the tube C H, which connects D with a stand pierced with an opening at H. On this stand is placed the vessel A (called the *receiver*), which it is required to exhaust. Suppose F to be close to the bottom of the cylinder, and to be raised. The valve C opening allows air to flow from A to D. When F descends the air in D is compressed, C closes, the valve F opens, air passes through it; and when F again reaches C, a quantity of air has been withdrawn from A equal to the content of D. F again rises, D is filled with air, and again emptied. The process goes on until after a few strokes the air in A is so rarefied that its pressure is no longer able to raise C. The action then ceases, and A is almost exhausted of air.

Fig. 146.

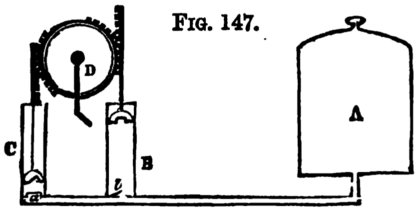


372. As the exhaustion proceeds the labour of working the pump increases. At first the pressure on both sides of the piston is the same, 15lbs. on the square inch. As the air in A becomes rarefied its pressure diminishes; so that, while the pressure on the upper surface remains the same, that on the under surface constantly decreases. Therefore, in raising the piston the labour constantly increases, until at last it reaches almost the atmospheric pressure of 15lbs. on the square inch, in addition to the friction.

373. **Hawksbee's Double-barrel Air-Pump** (Fig. 147).—There are here two pistons worked by means of racks, by the wheel D. They are so fixed that while B descends, C rises; the atmospheric pressure aids the descent of B, just as much as it opposes the ascent of C, so that the pressures on the pistons just balance. When B descends, air escapes through its valve; at the same time C rises, and air passes through *a* to fill the cylinder. When C descends *a* closes, air escapes

through the valve in C, b opens and air fills the cylinder B. The wheel has a reciprocating motion, so as to raise and depress the pistons alternately. The action ceases when the elasticity of the air in the vessel A is not sufficient to raise the valves b a. In the best modern forms of the air-pump the valves are opened by a mechanical contrivance independent of the elasticity of the air.

FIG. 147.



374. **Measure of the Degree of its Exhaustion.**—This depends on the ratio between the volume of the cylinder and that of the receiver and connecting pipe taken together. Taking, for simplicity, Smeaton's pump, let a be the volume of the receiver A and the tube H C, and b that of the cylinder. On raising the piston the air which before occupied volume a expands into volume $a+b$. At the next stroke a volume b of this expanded air is removed, and the volume a is left. This is $=\frac{a}{a+b}$ of the whole mass M of air at first in the receiver. After the second stroke the quantity left will be $\frac{a}{a+b}$ of this or $\left(\frac{a}{a+b}\right)^2$ of M. After any number of strokes, the quantity left will be $\left(\frac{a}{a+b}\right)^n$ of the whole original mass.

375. Thus if the volume of the cylinder be 10 cubic inches, of the receiver 100 cubic inches,

After first stroke the density will be $\frac{100}{110}=\frac{10}{11}$.
" second " " $\frac{100}{110}$ of $\frac{10}{11}=(\frac{10}{11})^2$.
" third " " $\frac{100}{110}$ of $(\frac{10}{11})^2=(\frac{10}{11})^3$.
" tenth " " . . . . $(\frac{10}{11})^{10}$.

If the volume of the receiver be three times that of the cylinder,

Density after first stroke $=\frac{3}{4}$.
" second " $=\frac{3}{4}$ of $\frac{3}{4}=\frac{9}{16}$.
" third " $=\frac{3}{4}$ of $\frac{9}{16}=\frac{27}{64}$.

376. **The Mercurial Gauge** (Fig. 148) is employed to show the degree of exhaustion produced in the receiver at any time. It consists of a bent tube, each leg of which is about a foot long, and which communicates freely with the receiver. One end, A, is closed, and the leg A B filled with mercury; the other end C is open. As soon as the pressure of the air in the receiver becomes insufficient to sustain a column of mercury of height A B, the mercury falls in A B, and rises in B C. The pressure of the air is then shown by the difference of level in the two tubes. Thus if the difference were one inch, while the barometer stood at 30 inches, the pressure of the air would be only $\frac{1}{30}$ of one atmosphere. The pressure on the receiver will always be half as many pounds per square inch as there are inches between the levels of the surfaces. If the air could be entirely exhausted, the mercury would stand at the same level in both branches. But this cannot possibly be the case. Sometimes the instrument is lengthened in the way shown by the dotted line, and screwed on at D.

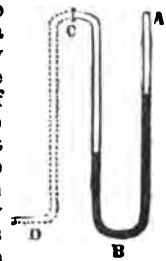


FIG. 148.

377. **The Hydraulic Press** (Fig. 149) depends on the law

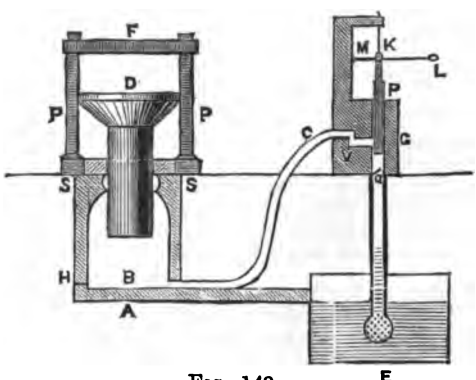


FIG. 149.

of equal transmission of pressure through liquids. It is

often called Bramah's Press, from Bramah, who first constructed it, 1796.

It consists of a cylinder, A, with very strong sides, a large cast-iron ram, B, which works water-tight in the top of the cylinder, and a force-pump, G. To the top of the ram is attached a strong plate, D, on which is placed the substance to be acted on. The plate F is supported by four strong pillars, two of which are here seen, P P. The cylinder is connected by a pipe, C, with the force-pump, G, wrought by the handle, L.

378. When the plunger of the pump is raised, the valve *a* opens, and after one or two strokes water rises through *a* from the cistern E. When P next descends, *a* closes, and the water is driven through the valve *v* into the cylinder. By the continued working of the pump water is constantly forced into A; and it cannot return because the valve *v* closes against it; and besides the pump is always full, so that there is no room in it for more water. The water not being compressible, room must be found for it in A. The ram, B, is the only movable part of the solid apparatus in A. It is therefore forced upward, and the substance on D is pressed between D and F with great force. By turning a handle at H, the water is allowed to flow through a pipe into the cistern, E, when B descends, and the compressed substance is removed.

379. The pressure on P, the plunger or piston of the pump, is to the pressure on B : : area of section of P : area of section of B. The mechanical effect of the press is therefore equal to $\frac{\text{area of B}}{\text{area of P}}$ or to $\frac{\text{diameter}^2\text{B}}{\text{diameter}^2\text{P}}$. The pressure exerted on the given substance is thus equal to the pressure on the plunger multiplied by the square of the diameter of the ram, and divided by the square of the diameter of the plunger. Thus, if the diameter of *p* be one inch, and of B 10 inches, and if a force of 120 lbs. work the pump, the pressure on the ram is $120 \times \frac{100}{1} = .12{,}000$ lbs.

The effect of the lever must be taken into account. The power is applied at right angles to the lever, and is increased by a factor $= \frac{\text{whole length of lever}}{\text{length of its shorter arm}}$.

380. The hydraulic press is used in compressing cloth and other substances, in pressing oil from seeds, in extracting the juice of beetroot, in stamping the covers of books, and

in raising great weights. The parts of the tubular bridge over the Menai Straits were lifted into position by a very powerful Bramah's press, capable of raising 2,000 tons weight. Its cylinder was 9 feet long, and 22 inches in diameter.

EXERCISES.

1. A cylindrical jar, 6 inches in height, full of air, is forced with its mouth downwards in water till the mouth is at a depth of 8 feet. How high will the water rise in the jar? *Ans.* 1·11 inch.

2. A conical wine-glass is immersed mouth downwards in water; how far must it be depressed that the water within the glass may rise half-way up in it? *Ans.* 238 feet.

3. A cylindrical diving-bell, 10 feet high, is sunk till the top of the bell is at a depth of 45 feet; find to what height the water rises in it. *Ans.* 5·91 feet.

4. Describe the suction pump, illustrating your answer by a sketch showing the action of the valves. How and why is the action of the suction pump limited?

5. Why cannot a lifting pump raise water from any depth? and what is the limit?

6. Describe the siphon, and how it is used. Why must the branch at which the fluid enters be shorter than that at which it comes out?

7. An air-pump is so constructed that at each stroke one-third of the content of the receiver is removed. If the air before the first stroke is under a pressure of 30 inches of mercury, what is its pressure after the third stroke? (Variation of temperature to be neglected.) *Ans.* 8$\frac{8}{9}$ inches.

8. Describe briefly the mercurial gauge used for measuring the degree of exhaustion of an air-pump.

9. In a hydraulic press the diameter of the plunger of the pump is 3 inches, that of the ram 20 inches. A force of 35lbs. is applied to work the pump. Find the pressure on the ram. *Ans.* 1555$\frac{5}{9}$lbs.

10. A hydraulic press is worked by a force of 10lbs. by means of a lever L M (Fig. 149), 3 feet long; the shorter arm, M K, being 9 inches. The diameter of the plunger of the pump is 2 inches, that of the ram 16 inches. Find the pressure on the ram. *Ans.* 1024lbs.

APPENDIX.

EXAMINATION PAPERS.

Science and Art Department.

SUBJECT VI.—THEORETICAL MECHANICS.

ELEMENTARY STAGE, 1881.

1. What is density, and how is it measured? What is meant when a rod and when a lamina (or thin plate) is said to be of uniform density? (10.)
2. If a number of equal triangular laminæ are on the same base and on the same side of it, their centres of gravity will be in the same straight line. Why is this, and where is the line situated? (10.)
3. If three forces act at a point and balance each other, how must they be adjusted if the one contains as many units of force as the other two together? How must forces of 7, 10, and 3 units be adjusted if they act at a point and are in equilibrium? (12.)
4. Define the centre of two parallel forces? Forces of 5 and 7 units act in the same direction along parallel lines at points 2 feet apart; where is their centre? If the direction of the former force is reversed where will now be their centre? Show the position of the centre in each case by a diagram. (12.)
5. In a single fixed pulley working without friction, the power is applied horizontally and the weight (of course) vertically; what is the relation between the power and the weight, and what the magnitude and direction of the force exerted on the beam which supports the machine—the weight of block, pulley, and rope being neglected? If we suppose the block containing the pulley to be suspended from the beam by a rope, show by a diagram the position in which it comes to rest. (14.)
6. A pumping engine is partly worked by a weight of 2 tons which at each stroke of the pump falls through 4 feet; the pump makes 10 strokes a minute; how many gallons of water (one gallon weighing 10 lbs.) are lifted per minute by the weight from a depth of 200 feet?

   N.B.—You need not concern yourself with the question how the weight is lifted between the strokes. (10.)
7. What is meant when it is said that the acceleration of the velocity of a particle is ten, the units being feet and seconds? If the particle were moving at any instant at the rate of $7\frac{1}{2}$ feet a second, after what time would its velocity be quadrupled? And what distance would it describe in that time? (14.)
8. Define a simple pendulum, a compound pendulum, and the centre of oscillation.

   A rod suspended by a fine axis passing through one end of it

makes 66 oscillations a minute; find another point in it from which if it be suspended it will still make 66 oscillations a minute ($g=32$). (12.)

9. Two equal small areas (A and B) are marked on a side of a reservoir at different depths below the surface of the water; what is the ratio of the pressure of the water on A to its pressure on B?

   The pressure on A is four times the pressure on B; but if the water is drawn off, so that the surface of the water in the reservoir falls a foot, the pressure on A is now five times the pressure on B; at what depth were A and B below the surface in the first instance? (12.)

10. Define the specific gravity (or specific density) of a liquid or solid.

    A rod of uniform cross section 18 inches long weighs 3 oz.; its specific gravity is 8·8; what fraction of a square inch is the area of its cross section? The weight of a cubic inch of water may be taken to equal 252 grains. (10.)

11. If you cut a slice off a cork by a plane parallel to the axis, and then put what is left of the cork into water; in what position will it float, and why?

    N.B.—You may assume that the cork is so long as not to float with its axis vertical. (14.)

12. A barometer stands at 30 ins. the vacuum above the mercury being perfect; the area of the cross section of the tube is a quarter of a square inch; if a quarter of a cubic inch of the external air is allowed to get into the barometer and the mercury is found to fall 4 ins.; what was the volume of the original vacuum? (12.)

ELEMENTARY STAGE, 1882.

1. Define Mass and Density. Of two bodies one has a volume of 5 cubic inches, the other of one-fifth of a cubic foot; in a perfectly just balance the former weighs 15 oz. the latter 12·8 lbs.; what is the ratio of the mass of the first to that of the second, and what is the ratio of the density of the first to that of the second? *Ans.* (1) 75 : 1024; (2) 81 : 16. (10.)
2. Where is the centre of gravity of a triangular piece of card board? Why should the centre of gravity of three equal particles placed one at each angular point of a triangle coincide with that of the triangle? (10.)
3. Define the resultant of two forces. Forces of 5 and 12 units respectively act at a point; what are their directions when their resultant is greatest, and what when it is least? Find the resultant when greatest, when least, and when the forces act at right angles to each other. *Ans.* 17, 7, 13. (12.)
4. Define the moment of a force with respect to a point. If two forces balance each other on a weightless rod capable of turning

freely round a fixed point, what relation must exist between the forces? C is a point in a weightless rod (A B) round which it is capable of turning freely; A C is one-third of A B; a force of 10 units acts at A perpendicularly to A B, and is balanced by an equal force acting at B; find how the second force must act, and the magnitude and direction of the pressure on the fulcrum. *Ans.* 10 lbs. at $\angle$ 30° with A B. R makes $\angle$ 60° with A B; and $R = \sqrt{300} = 17{\cdot}3205$. (14.)

5. A smooth heavy ring is threaded on to a fine cord, the ends of which are fastened to fixed points in the same horizontal line; in which position will the ring come to rest, and why? (10.)
6. What is a unit of work? In the single moveable pulley supported by parallel threads, show (*a*) that the weight equals twice the power, (*b*) that when the weight is lifted the work done by the power equals the work done against the weight. (10.)
7. A particle is found to be moving in a straight line at the rate of 5 feet a second; a quarter of a minute afterwards at the rate of 50 feet a second; half a minute afterwards, *i.e.*, after the first mentioned time, at the rate of 95 feet a second; show that this is consistent with the action of a constant force on the body, and find the ratio which that force bears to the force of gravity on the body.

   Ans. $f = 3$; $\dfrac{F}{W} = \dfrac{f}{g} = \dfrac{3}{32}$ (12.)
8. A body moves in a circle with a constant velocity, what force must act on it?

   A body whose mass is 10 lbs. is tied to a thread 6 feet long, and is allowed to swing backwards and forwards through the arc of a semicircle; when it is 30° from the lowest point of the arc, what forces are acting on the body?

   Ans. Since $v^2 = 2\ gr\ \dfrac{\sqrt{3}}{2}$, and $W = 10$, $F = 10\sqrt{3}$, but $T = F + W.\ \text{Cos}.\ 30° = 10\sqrt{3} + 5\sqrt{3} = 15\sqrt{3}$.
9. When a wheel rolls along a horizontal road, what are the velocities of the ends of the diameters that are at any instant vertical and horizontal? (14.)
10. A body whose weight is 12 lbs. and specific gravity 2·5 is placed in a vessel with a horizontal base containing water; what pressure is exerted on the base by the body, supposing the body to be quite covered with water? *Ans.* 7·2 lbs. (10.)
11. A hollow cylinder is full of air at a pressure of 15 lbs. per square inch when the piston is 12 in. from the bottom; if more air is forced in till there is three times as much air as at first, and if the piston is allowed to rise 4 in.; what is now the pressure of the air per square inch (temperature constant)? *Ans.* $33\frac{3}{4}$lbs. (12.)
12. Explain the action of the common suction pump, and find the force acting along the piston rod required to work it. (12.)

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